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HP PRIME

GRAPHING CALCULATOR COMPENDIUM

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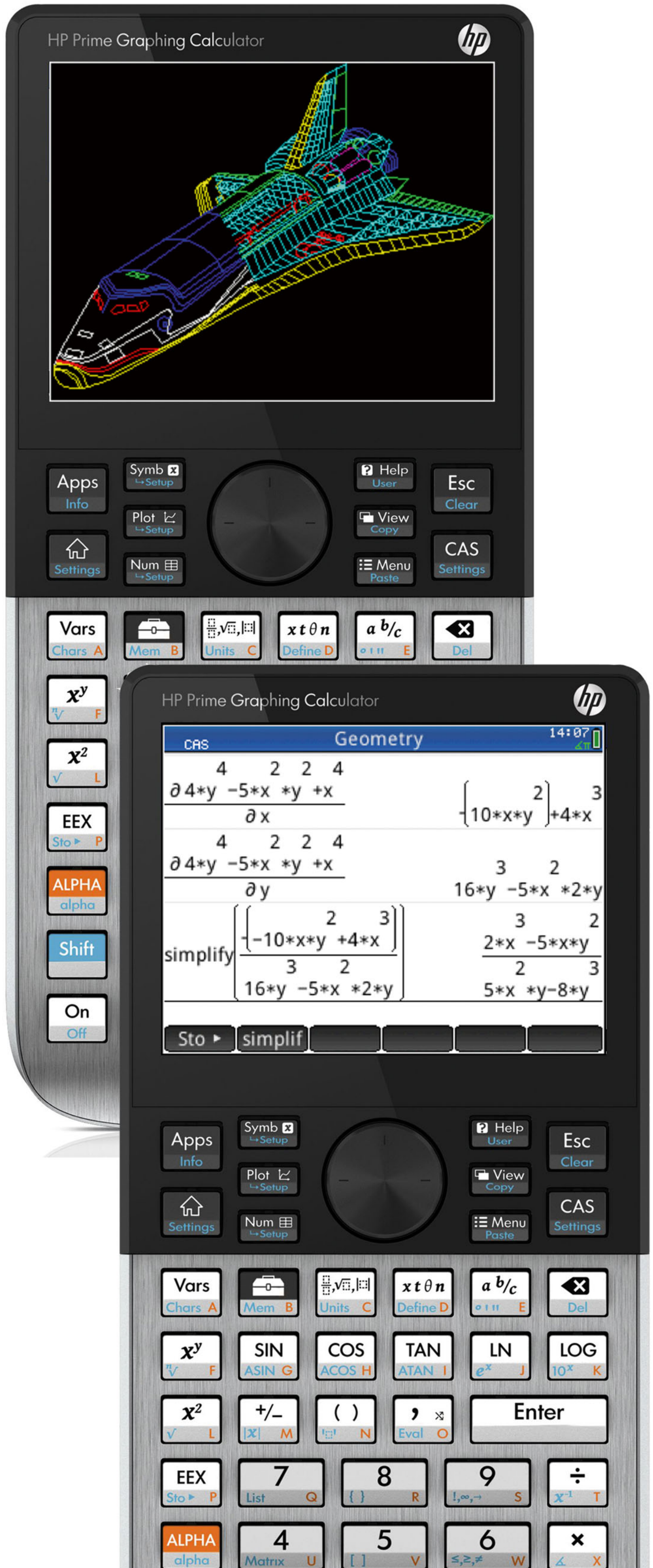
BEST BRANDS IN BEST QUALITY

HP PRIME GRAPHING CALCULATOR

HP-Prime
Touchfähiger grafischer CAS Grafikrechner.



- Mit dem HP Prime können Problemstellungen aus den verschiedensten Anwendungsbereichen bearbeitet werden.
  Dazu enthält der HP Prime u.a. ein leistungsstarkes CAS, eine funktionelle dynamische Geometrie-Software, eine praktische Tabellenkalkulation und eine Vielzahl von Applikationen.
- Flexibel und kommunikativ: Die Fähigkeit zur Kommunikation mit anderen HP Prime-Rechnern, das System für eine Messwerterfassung, der integrierte Prüfungsmodus und der HP Prime-Emulator für den PC bieten eine enorme Erweiterung der didaktischen Möglichkeiten.
- Tauch-fähiges 3.5 Zoll Farbdisplay, das auf Berührungen mit dem Finger reagiert, h. alle Funktionen des HP Prime können durch kurzes Antippen des Displays aktiviert werden. Damit präsentiert HP einen Taschenrechner mit einer komfortablen Bedienung, die vielen Menschen bereits durch Smartphones oder Tablets vertraut ist. Die Vorteile der Multi- TouchTechnologie zeigen sich in vielfältiger Weise:
- Graphen oder Diagramme können zur genaueren Analyse durch eine Fingergeste beliebig verschoben werden. Damit entfällt ein langwieriges Navigieren mittels Steuerungstasten.
- Durch Berühren des Displays können geometrische Konstruktionen wie auf einem Zeichenblatt erzeugt werden. Der Zugmodus kann direkt mit einer Fingerbewegung in alle Richtungen ausgeführt werden.
- Die Tasten des Taschenrechners werden jeweils durch passende Menüfelder ergänzt, die am unteren Displayrand kontextabhängig eingeblendet werden. Durch Antippen eines angezeigten Menüfeldes wird die dazugehörige Funktion ausgeführt.

Klares Bedienkonzept

Klares Bedienkonzept: Der HP Prime wurde mit einem klaren Bedienkonzept ausgestattet. Dieses besteht aus drei Grundkomponenten, die durch die Tasten Apps, Home und CAS ausgewählt werden können.

- Apps-Umgebung: In der Apps-Umgebung stehen eine Vielzahl von Applikationen zur Verfügung, mit denen sich Aufgabenstellungen aus den verschiedensten mathematischen und naturwissenschaftlichen Disziplinen visualisieren und lösen lassen.
  Alle Apps werden gleichermaßen über die Tasten Symb, Plot und Num gesteuert.
- Home-Umgebung: In der Home-Umgebung verhält sich der HP Prime wie ein gewöhnlicher Taschenrechner, d. h. die Ergebnisse werden numerisch berechnet.
- CAS-Umgebung: In der CAS-Umgebung ist das Computeralgebrasystem aktiv, d. h. alle Berechnungen werden symbolisch ausgeführt.
- Numerische Home-Umgebung: In der Home-Umgebung werden alle Zahlen numerisch interpretiert und alle Rechenoperationen numerisch ausgeführt.
  Für die Berechnungen stehen sowohl der „Textbuch"- Eingabemodus als auch der HP-typische „UPN"- Eingabemodus zur Verfügung.

HP PRIME GRAPHING CALCULATOR CONTENT

ACTIVITY PACKS

· Exploring IB Maths at Diploma Level
· Exploring Maths at Advanced Level
· The Maths Activities Pack: Using HP Prime
· Introduction to HP Prime
· HP Prime Workshop Materials

Exploring IB Maths

Activities for the HP Prime

Chris Olley

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Introduction

This book is aimed at students studying for the International Baccalaureate Diploma with advanced Maths using an HP Prime calculator. The aim is to provide a range of activities which will help you become a confident user of the calculator while developing your skills in different mathematical topics. This book is not a replacement for the manual, it is not the best way to quickly find out to do something specific. Use the manual or search the web. Also, there is no attempt to cover the entire course, however there is a good range of topics covered. The power of this technology is in its capacity to generate lots of mathematical information very quickly, so you can get a good feeling for mathematical ideas. Mathematics needs to be explored and with your HP Prime you can get under the skin of the ideas you need to learn about. I hope that you will try out the activities in this book and get into the spirit of exploration that you can then use in all of the topics you need to study.

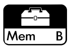
About the Author

Chris Olley was a secondary school maths teacher in a range of comprehensive schools in London and East Africa. He currently directs the secondary maths PGCE course at King's College London. He has worked with graphing calculators since they first arrived in the late 1980s and has run sessions nationally and internationally on different approaches to dynamic ICT in maths education, of which graphing calculators are an excellent example.

Further Information and Support

Please visit www.hpgraphingcalc.org and join in the discussion threads. Share your ideas and your understanding of mathematics with others.

1. Studying for the International Baccalaureate Diploma Maths with an HP Prime

The HP Prime is a very powerful calculator. It can find solutions to a vast range of mathematical problems in algebra, calculus, probability and statistics, complex numbers, matrices and much more. Press the Toolkit Key  on the HP Prime to see the menu of commands and get a feeling for how much this machine will do.

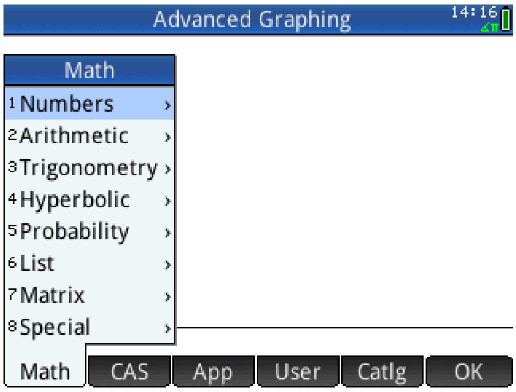
The big message for IB Diploma level maths students, is that you can use this machine in your exams (those for which a calculator is allowed). I would assume you have bought the calculator because you know this. It is really important that you do not think that the calculator will answer the exam questions for you. You actually have to do the steps in the process. However, the main source of difficulty in an advanced level exam is making small errors along the way. With this calculator you can quickly calculate the solution and check that you have done it correctly and be able to move on confidently. Knowing the answer first is often a very helpful way of deciding what steps to take. Also, being able to see different representations of an object quickly, helps you decide what the solution could be. With HP Prime you can draw graphs and see tables of values of different types of functions, which you can zoom in and out of, make calculations on, even do numeric integration and differentiation.

So, it won't tell you how to solve the problem, but will get you confidently to a solution with a range of better ways of seeing. HP Prime is a fantastically powerful tool and all of the evidence from different countries where graphing calculators are used in exams says that if you can use it properly, then it will give you an advantage.

However, it's not just in the exam that the calculator is useful. While you are learning maths, it is vital that you get a good intuitive feeling about how maths works. HP Prime is a fantastic tool for exploring mathematics. What happens to the graph of a quadratic $y = ax^2 + bx + c$ when you change the coefficients a, b and c? You would need to draw dozens of graphs to get a good feeling for what goes on. With HP Prime you can draw as many as you like, changing the coefficients selectively. Working this way helps you get out of the other big problem with advanced level exams. Memorising methods is fine until you find a question you can't recall the method for. That normally means most of them, because they never come up just like you were expecting. Far better is to understand what is going on and be able to see the mathematics from different starting points, then you can work your way through even when you can't decide which method to use. Maths is the method and you make yourself a mathematician by exploring. The HP Prime is the best tool to make that possible.

This book has a range of examples. Many of them will suggest areas of maths you can explore. But remember, they are just examples. Use the ideas to explore any new area of maths you are learning. By the time of your exam you will be so skilled in using the calculator that it will be able to support you quickly and powerfully in the exam itself.

2. The Home Screen and the Toolkit

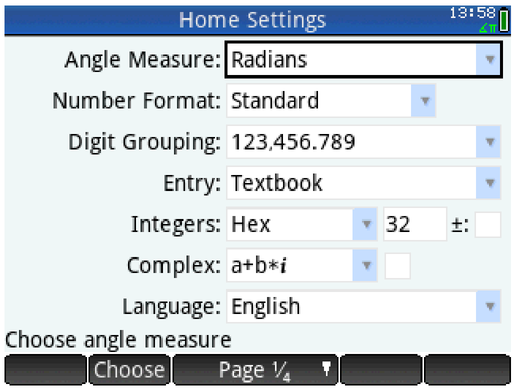


When you switch the calculator on, you would naturally start in the Home screen Settings. (Press this key to make sure you are there). Here you can do any type of calculation. Not just arithmetic, but using matrices, summing sequences, calculus, complex numbers and so on.

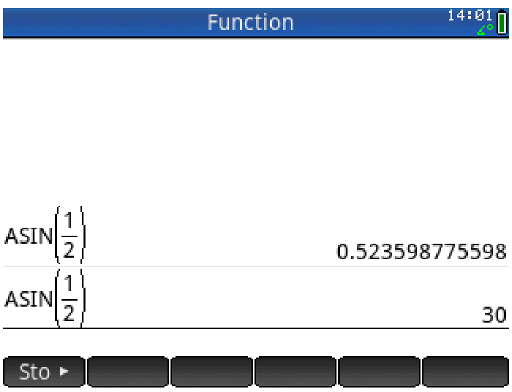
In fact there are two calculators. The Home calculator Settings and the CAS calculator CAS Settings. You can do calculations in either but there are some important differences. In general, if you want to find a numeric answer to a problem or you want an approximate (decimal) answer, then use the Home calculator Settings. If you want a symbolic (algebraic) answer or an exact (fraction) answer, then use the CAS calculator CAS Settings. First, we will look at the Home calculator Settings.

Pressing the Toolkit key Mem B gives the menu for all of the operations you can do on the Home screen. Notice the different menus available. To start with, look at the math menu (touch Math onscreen). This menu has several sub menus to explore. To find out how everything works, there is a comprehensive help system. Choose the function you want. It will be entered in the command line. Now press the Help key Help User to see what you need to enter and an example.

Using Numbers



You can decide how results are presented using Shift + Settings. Notably you can set the angle mode and the accuracy of results. Also, notice that you can change the language that the calculator works in.

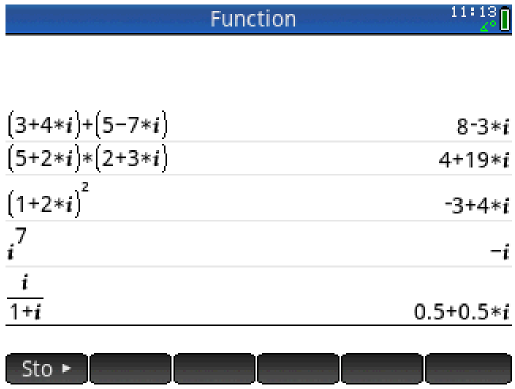


Calculate the inverse sin of $\frac{1}{2}$ in the default mode (radians) then change the setting to degrees. Tap your first entry and press copy, press the Enter ≈ again. It will be re-evaluated in Radians.

Experiment with different settings for accuracy. 'Fixed' allows you to set the number of decimal places.

You can also calculate with complex (imaginary) numbers.

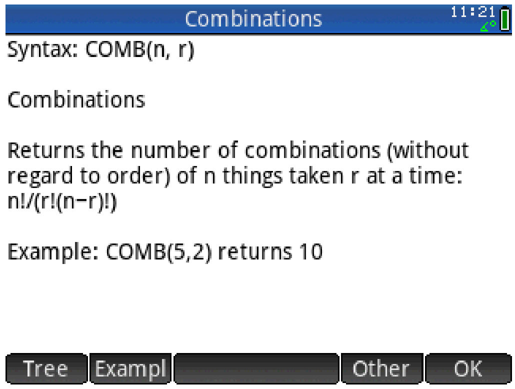
You enter i using Shift + 2



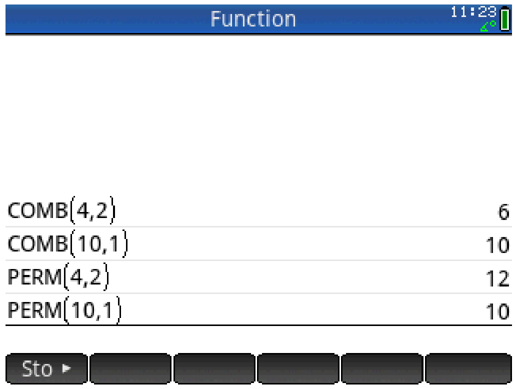
Enter $3 + 4i$ either directly or as (3,4). Decide which you prefer in the settings menu. Use brackets to make the calculation you intend clear.

Practice by entering a calculation, work out the answer you expect, then press Enter, to check.

If you want a mathematical function which is not on the keyboard, then you will find it with the toolkit key Mem. For example, permutations and combinations. Think about the category you would expect to find them under, or search by name alphabetically.



In the toolkit, follow Math, then Probability. Or use Catlg (catalogue) and press C (you do not need to press the Alpha key first in this menu) and navigate down. Press Help to tell you how the function works.



Work with the toolkit Mem and help Help to explore the vast range of functions available in HP Prime. In general the functions in the Math menu are most suitable in the Home calculator Settings and those in the CAS menu in the CAS calculator CAS Settings (see below). However, many work in both, so you should not restrict your exploration!

If you are doing Physics and/or Chemistry as well, then you will be tempted to check the Units and Const menus as well. Press Units using shift + template Shift + Units to find both menus. Here, you will find a comprehensive set of important constants that scientists need to know. Also, if you make calculations with units, the answer will be given in the correct units. This is very powerful for scientific work. Explore the possibilities using compound units that you know. Again, enter a calculation, decide what you think the answer should be, then press Enter Enter.

Template Entry

To enter a number of important and popular functions you can use the template key

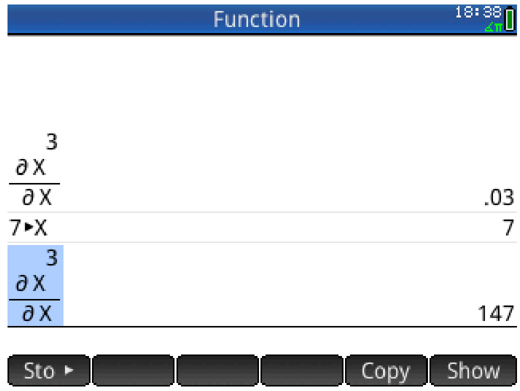
Let's calculate the differential of x^3 at $x = 7$. We know the differential is $3x^2$, so at $x = 7$ the result should be $3 \times 7^2 = 147$.

Choose the differential. Use $xt\theta n$ and the x^y key to enter the expression then press Enter.

You may get an odd answer here because X is a variable value and works as a memory, so it will have been evaluated with whatever value X has.

We wanted X=7, so we must say so. Enter 7 and press the Sto button on the screen and Enter.

Now press on the differential you entered before and press Copy (on screen), then Enter. Now we have evaluated the differential at X=7



Now we get the answer we were expecting.

On the HP Prime there are templates for many mathematical functions. You should explore them now and see that you can make them work as you would expect.

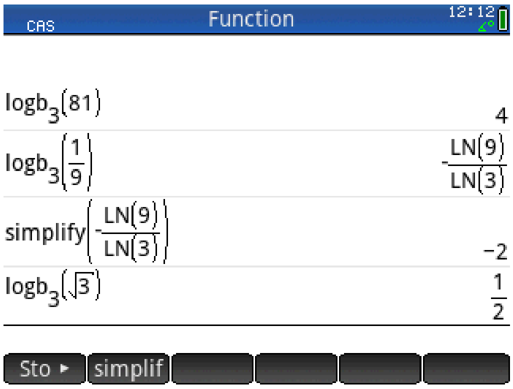
3. The Computer Algebra System (CAS)

You have been using the home screen which is the basic calculating screen. However, HP Prime has another calculating screen which looks almost identical. Press the CAS key to see. Notice in the top left hand corner of the screen you will see CAS. Otherwise it looks just the same.

Number Calculations

Use the CAS when you want to work with symbolic algebra or when you want an exact answer. The CAS will naturally report answers to calculations as fractions. A good example where this makes a difference is working with logarithms. With the template key choose the logarithm template.

Enter $\log_3 81$ and press Enter.



Now try $\log_3 \frac{1}{9}$. You were expecting the answer -2 but instead you get $-\frac{\text{LN}(9)}{\text{LN}(3)}$. This is because CAS shows you the steps in the calculation. So, you can see it is using the change of base formula. This can be very useful in exploring how things work. However, if you just want the answer, press simplif (for simplify).

Now try $\log_3 \sqrt{3}$. You should know what outcome to expect. Practice with logarithms with different bases.

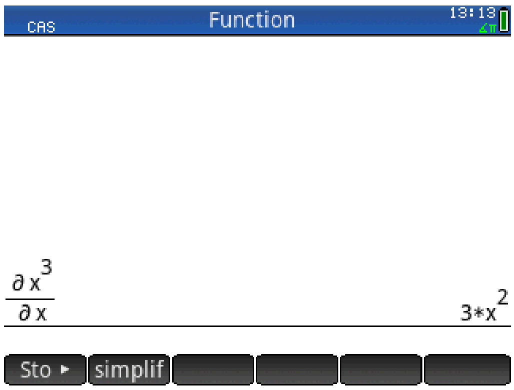
Try these again in the Home calculator to see the difference. Also, look for the LOG command in the toolkit.

Note: you can change how much the CAS simplifies when your press Enter in the CAS settings Shift + CAS

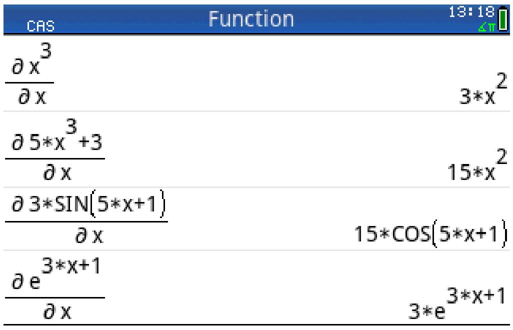
Symbolic Operations

On the home screen we calculate the differential of x^3. Do this again in the CAS.

Make sure that CAS is showing in the top left corner.



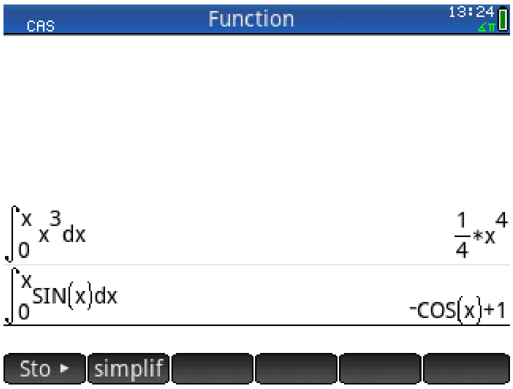
As before, use the template key and choose the differential. Use and the x^y key to enter the expression then press. Notice that enters a lower case x. This is very important. In CAS all variables are lower case. This distinguishes them from the use of letters in the Home screen, where letters are unknowns, which can be given values.



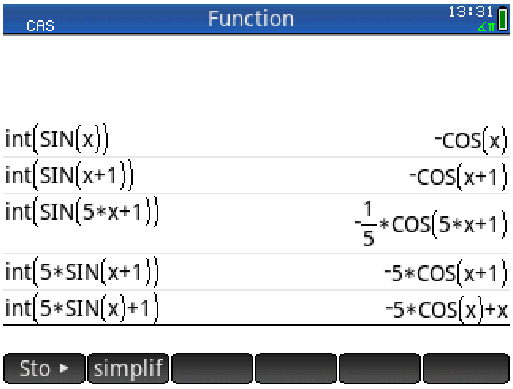
In CAS the differential is evaluated symbolically.

You can now explore the differentials of a range of functions and investigate the differences made by changing functions.

Using the template key you can explore integrals too. However, you need to set limits (from 0 up to X) which works fine for polynomials, but needs some interpretation for other functions.



Much better is to find the functions directly and enter the correct parameters. Explore the range of functions available in the toolkit CAS menu. When you select a function, press to get details on how the function works. Using the int() functions allows you to integrate directly (but you must include the +C in your written answer).



You should explore the integrals of functions. See what effect making slight changes to the functions makes.

CAS is an extremely powerful tool for exploration and you should explore any new functions you are studying in this way.

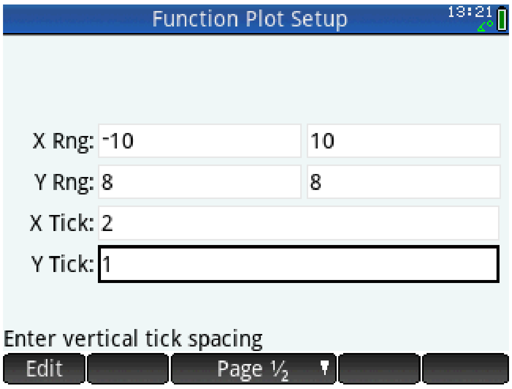
4. Working with Apps

The home screen is where you do calculations. You can always get back to the home screen by pressing the Home button . Pretty much everything else happens through Apps. Press the Apps key and scroll down the list. You will see most of your advanced level topics are covered. All of the Apps work the same way. The **Symb** **Plot** and **Num** keys show you the three representations of all the maths you can explore with Apps. **Symb** is for the algebraic view, **Plot** shows the graph view and **Num** shows the numeric or table view. Different Apps start in different views according to the maths, for example graphing Apps start in **Symb** mode, so you can enter a function, but Statistics Apps start in **Num** mode, so you can enter the data. The bar at the top of the screen tells you which App is running at any time. If you start up a new App, all of your data from the last App you were using is saved automatically and you can come back to it any time.

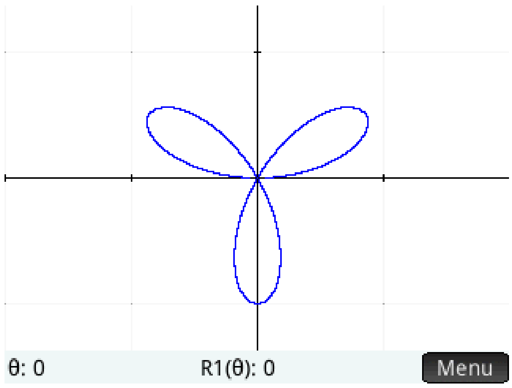
There are lots of settings/setup screens, so you need to decide what you want to set up.

The Home and CAS screens have their own settings for operations affecting the whole calculator. The settings for any App can be found with Symb Plot and Num . These settings only apply within the App you are running (shown in the bar at the top of the screen).

For example, if you want to fix the size of the axes on a function plot. This is a change to the Plot. So, press Shift + Plot . Notice that there are two pages of settings. On page 2 you can turn the axes and the grid on and off.



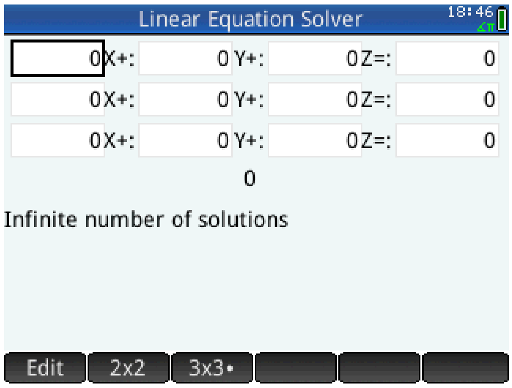
Finally, you can change the way the screen is organised using the **View** key. For example, you can split the screen and see the graph and a table of values at the same time.



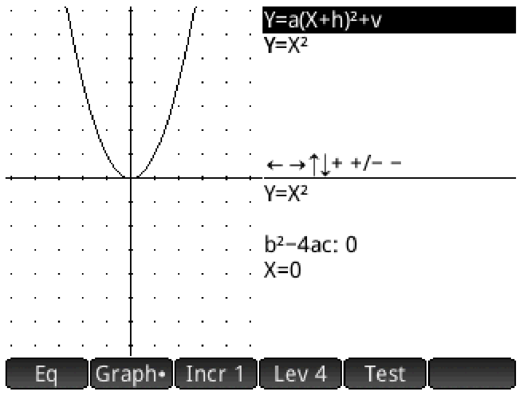
There are four main types of Apps:

1. **Function Apps (Blue)**. Here you enter the data or the function without restriction. (Function, Parametric, Polar and Sequence)

2. **Solver Apps (Orange)**. Here you get fast solutions to specifics types of problems. (Solve, Linear Solver, Triangle Solver and Finance). You should particularly explore the finance App which has built in functions for interest and depreciation. Explore using them, then work out how they work and do the calculations directly. You can use the app to check you have been successful.



3. **Explorer Apps (Green)**. Here you get a pre-configured set up to make exploring specific situation easy. These also contain tests to check your understanding. (Quadratic Explorer, Trig Explorer and Linear Explorer)



4. The **Advanced Apps**: Advanced Graphing, Geometry and Spreadsheet.

Hints and Tips

- If you are stuck on a menu press the Esc key to cancel the menu or the setting.
- Keep looking at the Help screens to see how things work.
- You can save apps with your settings preserved and attach notes to remind yourself what you were doing. Press SHIFT + Notes + to add notes. Then select the App **using the cursor** in the Apps menu. Click Save and give it a new name. You can then share this with anyone else, or indeed online.
- Install the connectivity software onto your computer. This way, the main operating system of the calculator can be updated when changes become available, you can upload and download Apps and you can type notes for your Apps or Programme your calculator much more efficiently with a keyboard!
- HP calculator users are a great enthusiast community and there are additional Apps created and being created for you to download and add to the functionality of your machine. So, check regularly to see what is available, and of course, contribute yourself. Check at:
    - www.hpgraphingcalc.org which is a general purpose support site
    - www.calc-bank.com which has programmes and activities

5. Graphs and Functions

You will need to explore different families of functions:

Polynomial functions
$f(x) = ax + b$
$f(x) = ax^2 + bx + c$
$f(x) = ax^3 + bx^2 + cx + d$
(etc.)

Trigonometric functions
$f(x) = \sin(x)$
$f(x) = \cos(x)$
$f(x) = \tan(x)$

Reciprocal Functions
$f(x) = \frac{1}{x}$

Exponential Functions
$f(x) = a^x$

Exploring Linear Functions

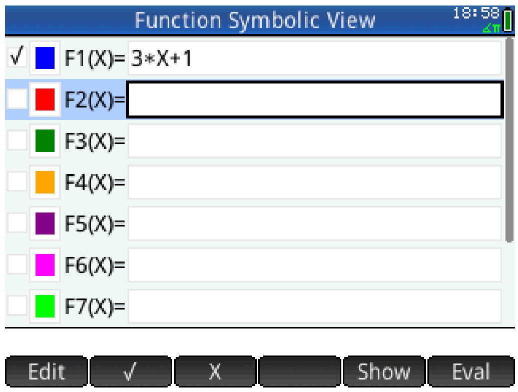
It is very important that you are able to look at a function and have a good feel for the size, shape and position of the graph of that function. Your HP Prime is the perfect tool for exploring functions.

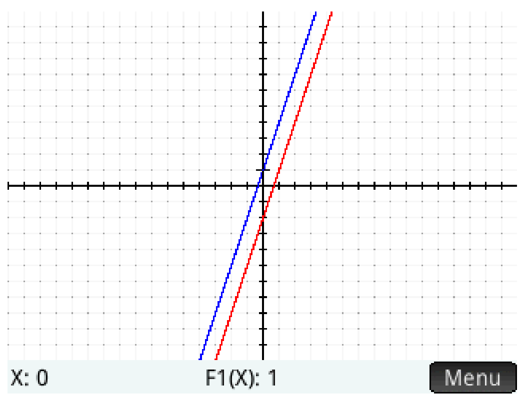
In the first instance you should do this for functions of the form $y = f(x)$ using the Function App. Later on you may need to do the same for polar functions, parametric functions and sequence functions, depending on the topics you are studying. There are Apps for all of these, but for now, let's focus on the Function App. Press the Apps Info key, choose Function, press RESET and OK to confirm, then START. (This makes sure that previous settings are cleared).

You should be very familiar with (linear) functions of the form $f(x) = mx + c$. You will know that these are straight line graphs and that they have a gradient of m and they cross the y-axis at $(0, c)$ which is called the y-intercept.

First you should make sure you know the relationship between the gradients of linear functions whose graphs are:

- Parallel
- Perpendicular





Your calculator is ready to receive an input for a function $F1(x)$, so try a simple Linear Function, say $F1(x) = 3x + 1$

Just type 3, press on screen for the X (or press the $x\,t\,\theta\,n$ Define key or press ALPHA alpha + ×), then +, then 1, then press OK.

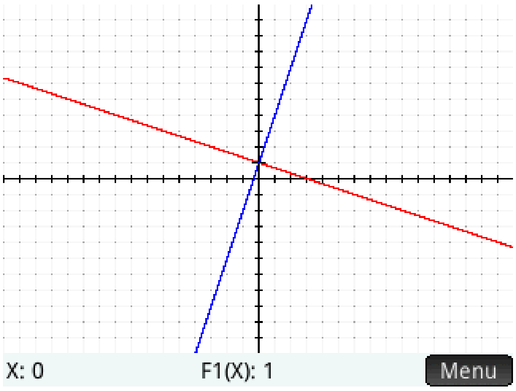
(Notice that the F1 now has a tick next to it, which means that when we choose Plot or Num modes, F1 will show up. You can touch the tick ✓ to turn it on and off).

Try a function for F2 which will be parallel to F1. The cursor has already moved down to F2, so you can type this directly. Now press the Plot key to see if you are correct.

Press the Symb Setup key, make sure the cursor is on the F2 line and try a new function for F2. Keep trying different functions until you are sure you know the relationship between the gradients.

Now do the same thing for Linear functions whose graphs are perpendicular.

I'm sure you knew the relationships already. But this will have helped you get used to entering and changing functions and looking at their graphs.



Quadratic Functions

Next, you should explore Quadratic functions. There are three standard ways of expressing a quadratic function:

1. Polynomial: $f(x) = ax^2 + bx + c$
2. Factorised: $f(x) = (x + a)(x + b)$
3. Completed square: $f(x) = (ax + b)^2 + c$

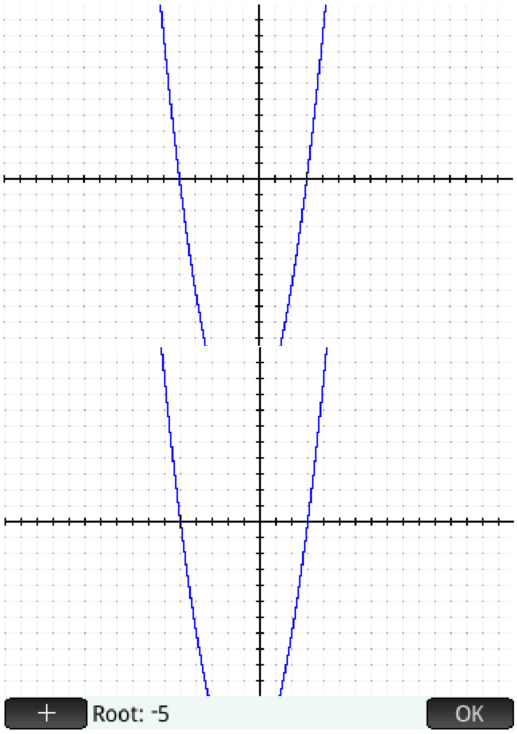
The first one is most familiar, but the factorised form gives the most information quickly (if the quadratic can be factorised). The completed square form probably gives the most complete picture, but the algebra involved in changing to this form is trickier. The important thing is that all three give different insights into the nature of the function. Your calculator will can do the algebra for you (using CAS), but you should practice converting between these three forms with pencil and paper.

Press Apps Info, select Function, then Reset/Ok/Start and enter a function F1 in factorised form, say $F1(x) = (x - 3)(x + 5)$ press OK, then press Plot.

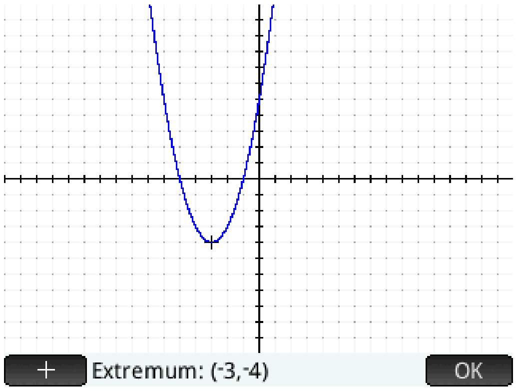
On screen, press Menu, then Fcn and select either Root by pressing near it on screen.

Now move the cursor so it is closer to the second root (or touching near it on screen). Now press Menu/Fcn/Root again.

The relationship between the roots and the factorised form is pretty clear. Try a few more examples to make sure.



Now get a feel for the completed square form. Replace your F1 with a function in completed square form, say $F1(x) = (x+3)^2 - 4$ (use the x^2 key to enter the 'squared'). Now look at the position of the minimum point with Menu/Fcn/Extremum.
Also, check the positions of the roots as before.

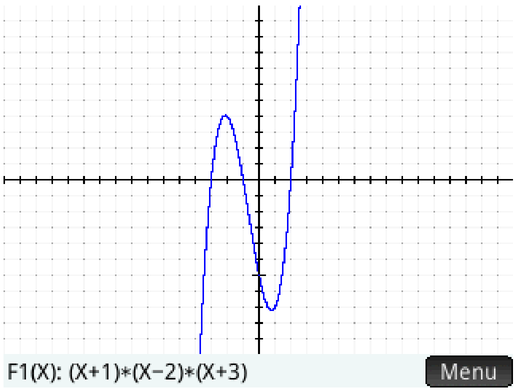


You will need to do some exploration before you can be sure of the relationship between the roots, the extremum and the values of a, b and c in $f(x) = (ax+b)^2 + c$, so keep going until you are quite sure. Then test your theory on new functions in this form.

Now you should be ready to take on the task of exploring the effect of changing a, b and c in the form $f(x) = ax^2 + bx + c$. Work on each one independently. The effect of changing the c is quite straightforward. The effect of changing the a is a bit counter-intuitive. However, just changing the b on its own is quite hard to describe. So, spend some time exploring until you are quite sure you have a good theory that you can describe accurately and simply.

Polynomials

Now you should explore other functions. Firstly, get a feel for cubic functions. Remember that different algebraic forms give different insights into the nature of the function. It is quite hard to imagine the graph of a cubic in the form $f(x) = ax^3 + bx^2 + cx + d$ but much easier in the form $f(x) = (x+a)(x+b)(x+c)$, so factorising first, if possible, is always a good thing.

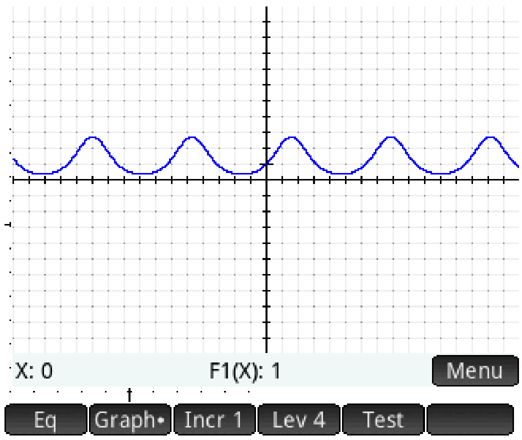


Try some quartic and quintic functions as well, to get a feeling for the differences between different polynomial functions.

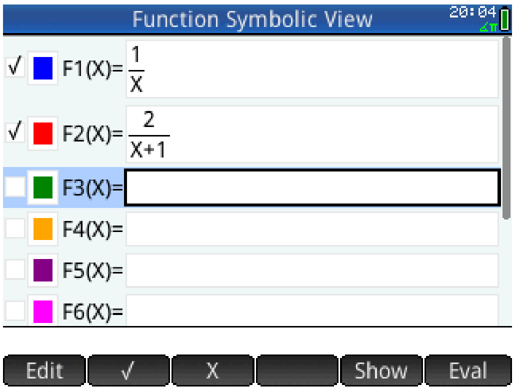
Other Functions

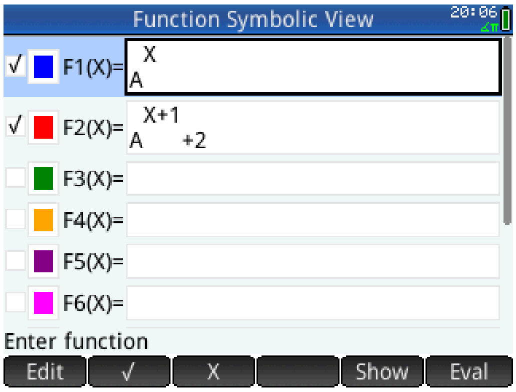
You should explore rational/reciprocal, trigonometric and exponential functions in the same way. Then explore composite functions (like $f(x) = e^{\sin(x)}$) and the inverses of functions.

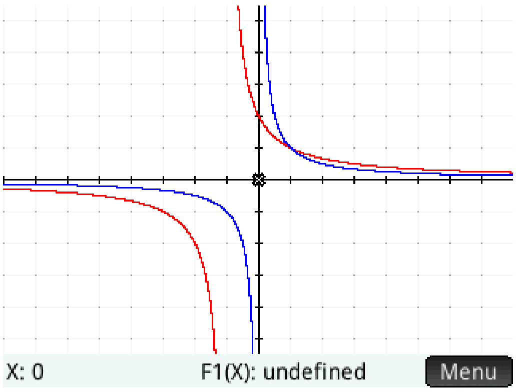
When you have a good feel for the general shape and position of these functions, you can explore how the graphs of functions can be manipulated generally.

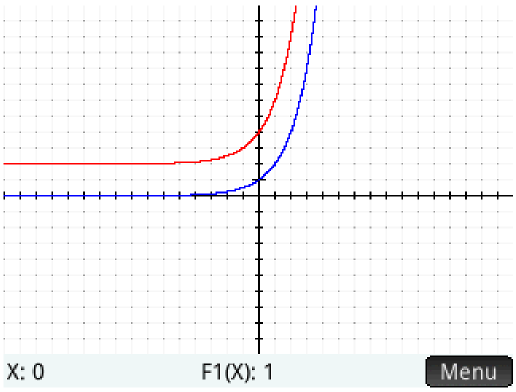


For example, start with a reciprocal function $f(x) = \frac{1}{x}$ and ensure that you can translate and stretch it horizontally and vertically. Repeat with an exponential function $f(x) = a^x$ and use HOME/Sto/ALPHA/A to change the value of A.









Using the Explorer Apps

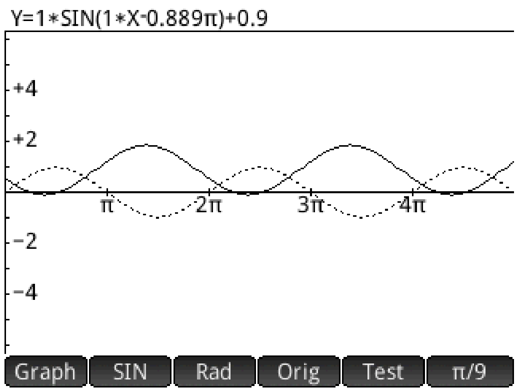
The Explorer Apps are great to test yourself when you feel ready. You only have Explorer Apps for Linear, Quadratic and Trigonometric functions, but that will be good to test your understanding of the transformations of graphs.

Press Apps Info and select Quadratic Explorer.

Use the cursor to move the graphs up, down, left and right and spread the graph in and out with the + and – keys. Look at the effect this has on the function.

Work through the 4 levels of difficulty (LEV1 to LEV4) and TEST yourself.

Now do the same with the Trig Explorer applet.



Piecewise functions

We can graph piecewise functions i.e. ones that are defined differently over different parts of their domain.

For example, we can graph a function for which $f(x) = 2x$ when $x < 1$ and $f(x) = 2$ when $x \geq 1$

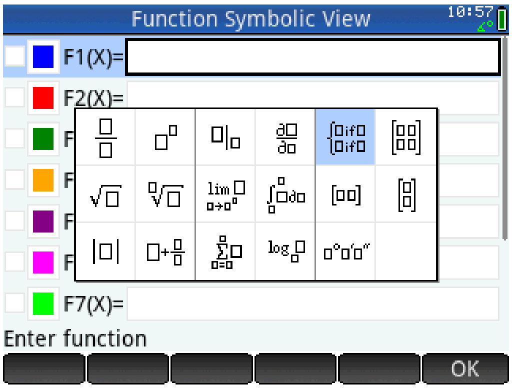
We write this function as $f(x) = \begin{cases} 2x & x < 1 \\ 2 & x \geq 1 \end{cases}$

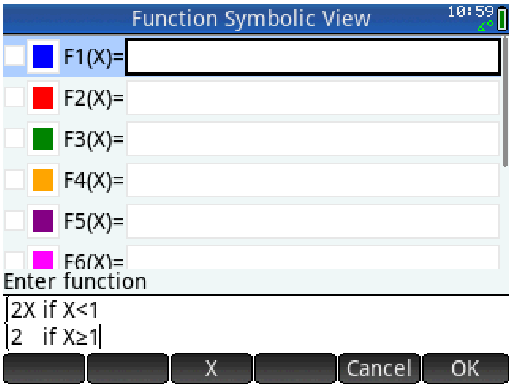
Press Apps and launch the Function applet.

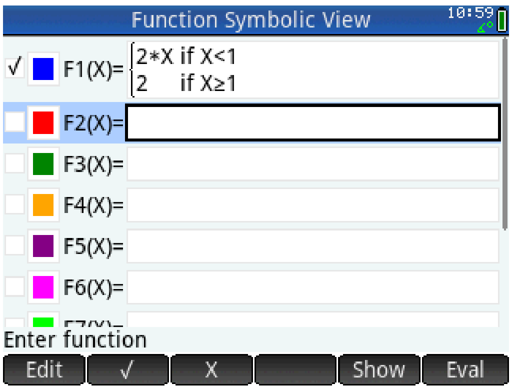
(Reset/OK/Start)

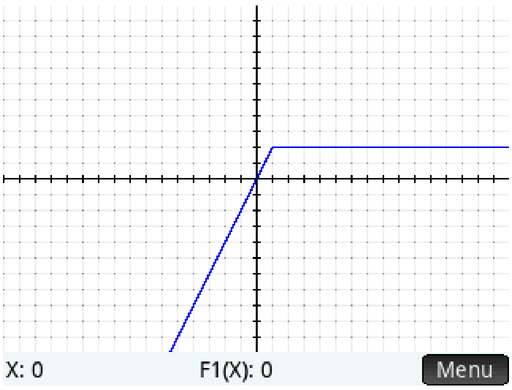
For F1(X) you should use the template menu Units and enter 2X then the domain X<2 using the menu with Shift + Units then enter 2 with domain x≥1 and Enter

Finally press Plot to see the effect. You should explore setting up different piecewise functions.









To set up piecewise functions with more than 2 parts needs the PIECEWISE() function. However this can only be used if the calculator is in algebraic entry mode. Go to settings Settings and on page 1 change the Entry mode to Algebraic (by default it is set to 'Textbook'). Now go back to the Function App Symb page and type e.g. PIECEWISE(X,X<1,2X,X=1,3X,X>1). It works with up to 8 parts like this.

To save typing, you can use the toolkit menu Mem and navigate to the PIECEWISE function.

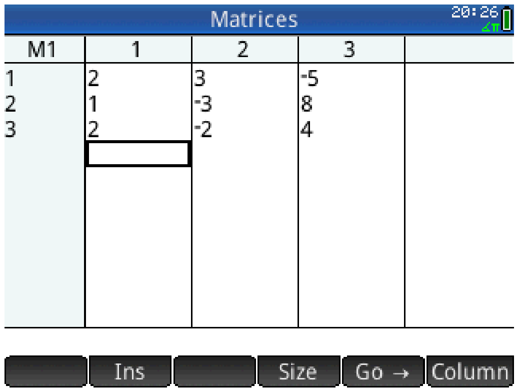
Now you can make some very exciting multi-part functions.

6. Solving Simultaneous Equation: Using Matrices

Mainly you will be using matrices for Linear Algebra. The Matrix Editor is opened with Shift + 4 Matrix. Here you can create Matrices in a convenient way. You can also type matrices directly onto the Home screen. When you have created a matrix, you should store it as a variable like M1, M2 etc.

Let's do a task. Solve the linear system:

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 &= 13 \\ x_1 - 3x_2 + 8x_3 &= -13 \\ 2x_1 - 2x_2 + 4x_3 &= -6 \end{aligned}$$

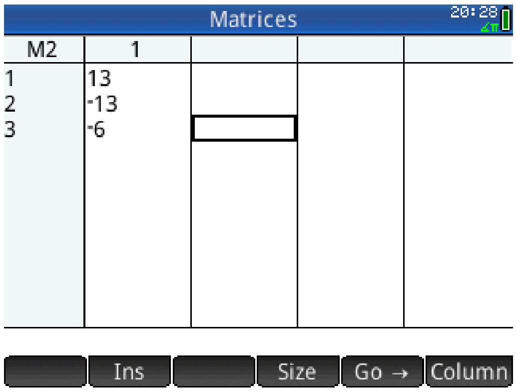


Use the Matrix editor to enter the matrix of coefficients.

This matrix is already called M1 by the editor. Just press Edit to get started and use the cursor keys and OK to enter each value. The Editor quickly works out the size of the matrix. When you are finished, press Home.

To solve the problem above we pre-multiply the solution matrix, by the inverse of the coefficients matrix. So, create a second matrix M2 with the right hand sides of the equations. This is a column matrix.

Use the matrix editor to enter M2.

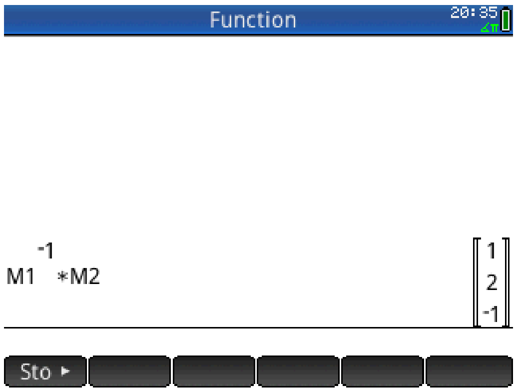


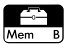
Now we are ready to find the solutions …

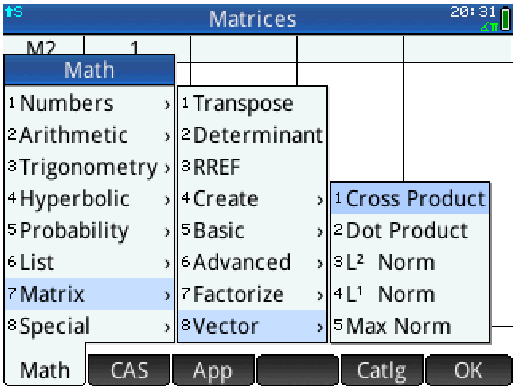
On the Home screen type $\mathrm{M1}^{-1} \times \mathrm{M2}$

(use the x^y key x^y and type -1)

So, we have solutions: $x_1 = 1, x_2 = 2, x_3 = -1$



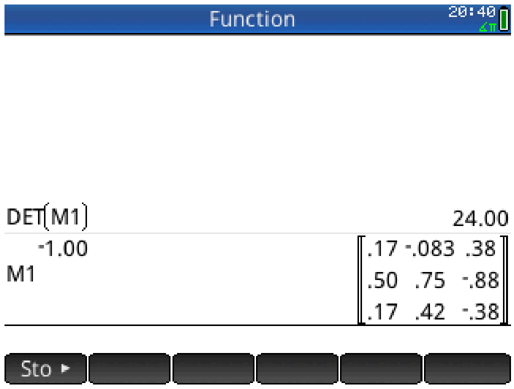
Press the Toolkit key  to find the Matrix section of the Math menu to see the range of operations we can apply to our matrix.



Entering the matrices can be a bit time consuming, but when they are entered you have an enormous amount that you can do with the matrices. So, now explore the other matrix functions available on the Math menu.

You can find the determinant.

You can find inverse matrices (notice that M2 has no inverse, so will generate a syntax error if you try this).



HP Prime shows the matrix in textbook format by default.

You can find Eigen Values and Vectors and do row and column operations. You should experiment to get a feel for all of the available functions and operations.

7. Sequences and Series

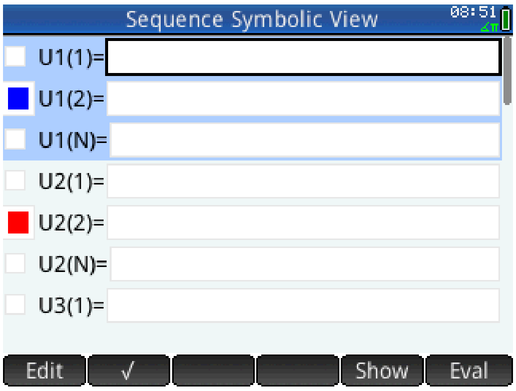
You will be very familiar with different continuous functions. You will have explored the relationship between the function, its graph and tables of values. With your HP Prime you can get a feeling for the variation in functions which are not continuous, by exploring sequences and series.

Look at a simple sequence: 3, 5, 7, 9, 11, 13, …

The first term is 3 and the rule is that each number is 2 more than the previous number (we have $u_1 = 3$ and $u_n = u_{n-1} + 2$).

(You may know this as an arithmetic progression with $a = 1$ and $d = 2$).

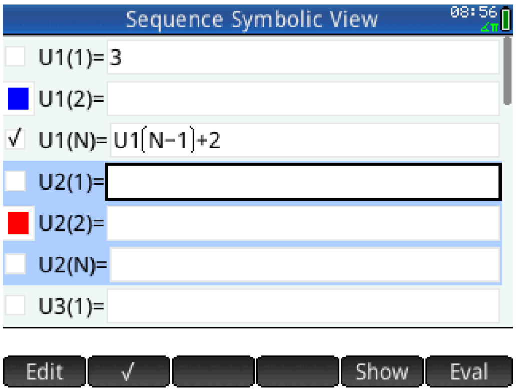
Let's look at this with the sequence app.



Press Apps Info, select Sequence then Reset/OK/Start

U1 is a sequence. We can enter its first term U1(1), its second term U1(2) and a rule for the nth term. You only need to enter what is needed. The first and second terms are not necessarily needed. In our case, we have a first term and a rule.

Enter the first term U1(1)=3

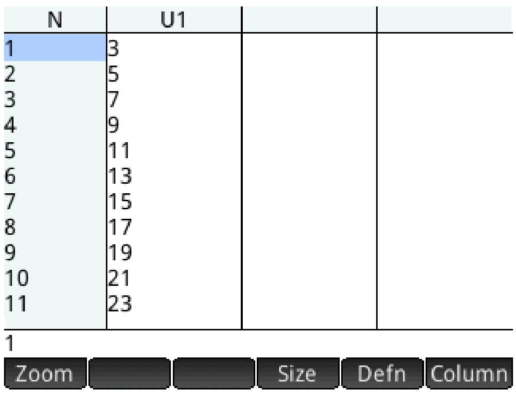


Then navigate down to U1(N). Notice that onscreen options have been added including (N-1) which we will need.

Use this to enter $u_n = u_{n-1} + 2$ as U1(N)=U1(N-1)+2

Press Num Setup to see the sequence.

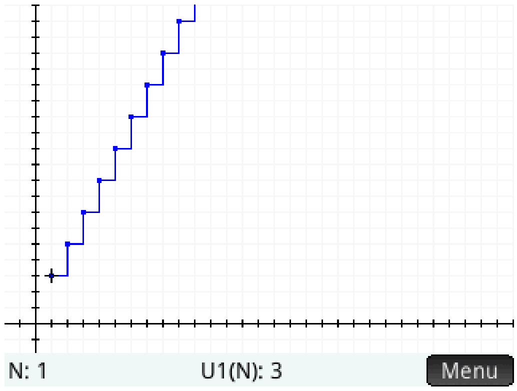
I pressed F4 to turn off the BIG font so you can see more of the sequence.



Use the down arrow to see even more.

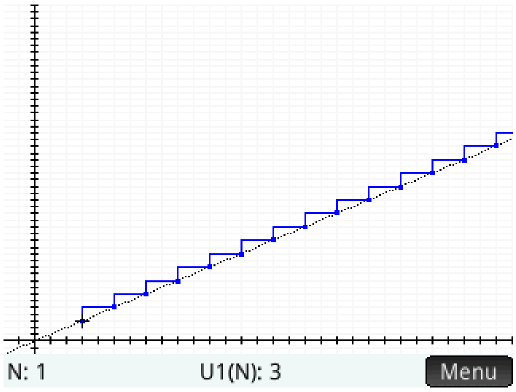
Notice that using the up arrow will not allow us to go back further than the first term, because we entered a value for U1(1) saying that this is where the sequence starts.

Now press Plot Setup to see what the sequence looks like graphically.



Press Shift + Plot Setup to see the plot setup options.

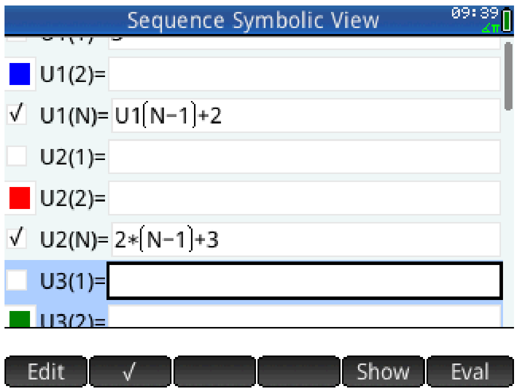
I have changed the plot type to Cobweb and increased the end of the Y range (YRNG) to 50 to see more of the sequence.



We have specified the sequence with a term-to-term rule. Now we can compare with a position to term rule, i.e. a rule for the nth term.

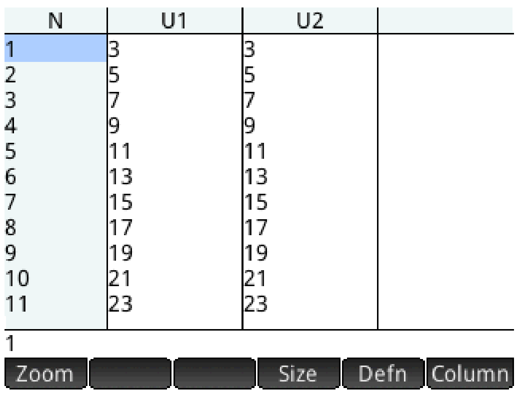
The first term is three and the sequence is increasing by two for each extra position so the position-to-term rule must be $u(n) = 2(n-1)+3$. Notice it must be $n-1$ because the first term has zero 2s added on.

Press Symb Setup and enter this rule as the second sequence U2. Enter the rule as U2(N)=2(N-1)+3



Notice that when you do this U2(1) and U2(2) are greyed, because with a position-to-term rule, any value can be calculated, so the initial terms are irrelevant.

Press Num Setup and compare the sequences.



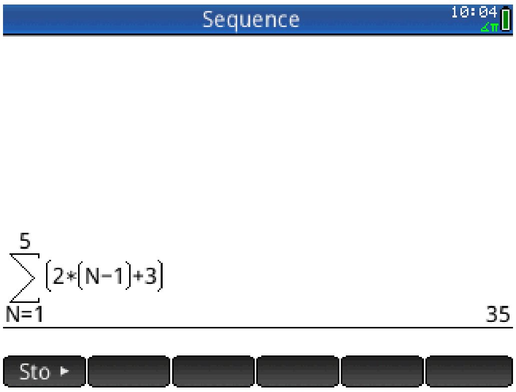
N	U1	U2
1	3	3
2	5	5
3	7	7
4	9	9
5	11	11
6	13	13
7	15	15
8	17	17
9	19	19
10	21	21
11	23	23

This provides an excellent platform to explore different sequences and practice finding position-to-term rules to match term-to-term rules.

Now that we know the position-to-term rule, we can find values for the corresponding series.

To find 3+5+7+9+11, we are summing terms in the sequence from $n = 1$ to $n = 5$

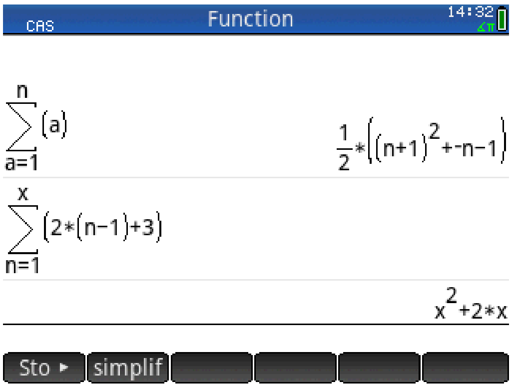
So, we find $\sum_1^5 2(n-1)+3$



Use the template key and choose the sum template.

Enter the values using ALPHA to enter the N.

Check that you agree with the result! (3+5+7+9+11 = 35)



You can easily change the values by using the cursor to select the entry, then pressing Copy and editing. You should explore the effect of making changes to the limits and to the rule.

Explore the $\sum$ template in CAS. Use a variable to sum up to.

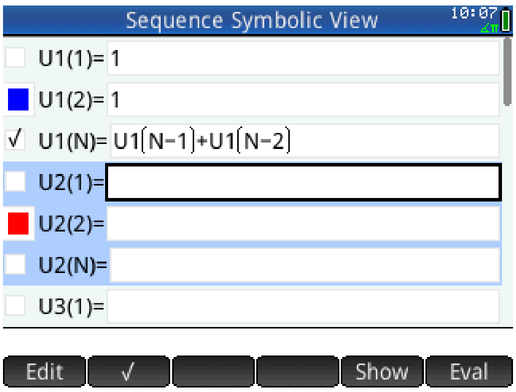
You can easily explore more complicated sequences. An excellent example would be Fibonacci sequences.

1, 1, 2, 3, 5, 8, 13, …

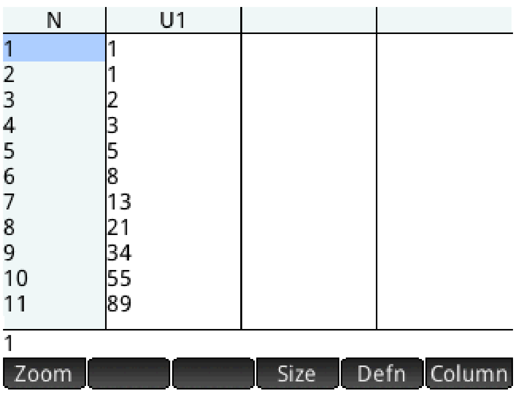
Here, we know the first and second terms and a rule that each term is the sum of the preceding two.

Relaunch the sequence applet and reset it, ready to enter the new sequence.

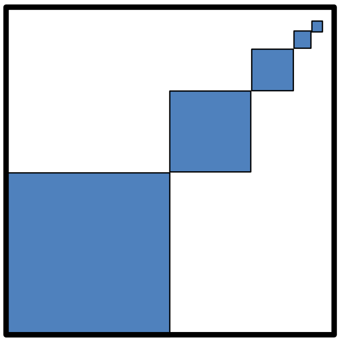
We have U1(1)=1, U1(2)=1 and U1(N)=U1(N-1)+U1(N-2)



Check that it has worked by looking at the table by pressing Num Setup.



Squares into Quarters: A Series Exploration



A square is divided into quarters and one quarter is shaded. A quarter of the opposite unshaded quarter is shaded. A quarter of this quarter is shaded and so on as shown. What fraction of the original square is shaded?

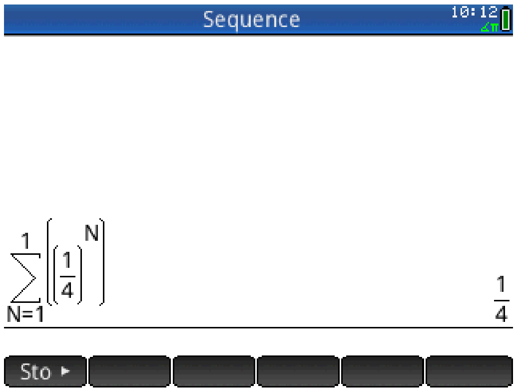
If the original square has unit area, then the shaded part is $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots$

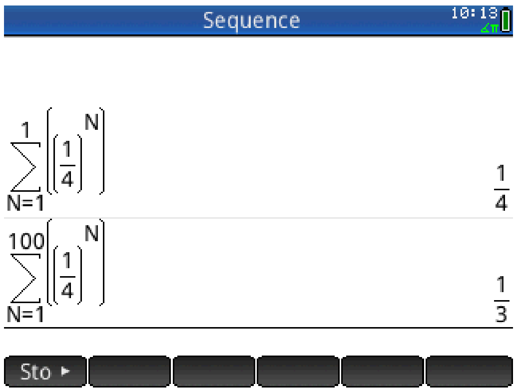
That is, adding up the terms of the sequence with $u_n = \left(\frac{1}{4}\right)^n$

So, see what happens to this value as n increases.

Enter $\sum_1^1 \left(\frac{1}{4}\right)^n$

Press *a*↔*b*/*c* to see the outcome as a fraction.





Use the up arrow to COPY and EDIT the sum. Keep increasing the last term to see how the sum is changing. What value does it tend towards as the number of terms increases?

Try to explain how you know that the shaded area must be this fraction of the whole.

8. Statistics: Probability Distributions and Inference

For statistical work, you should use the three statistics applets:

- 1 variable statistics (for box plots, histograms etc. with central tendency and spread)
- 2 variable statistics (for scatter plots, correlation and regression)
- Inference (for inferential statistics and confidence intervals)

Also, press Math and look at the following sub menus:

- Distribution (binomial, normal, poisson, etc.)
- Probability (permutations, combinations, random numbers, etc.)

One Variable Data

Get started by entering some data for 2 single variable lists.

Apps/Statistics 1Var/Reset/OK/Start

For example, the ages of visitors to a swimming pool on Friday are as follows:

{12, 13, 12, 12, 12, 12, 3, 38, 12, 13, 12, 13, 12, 12, 12, 13} enter this under D1

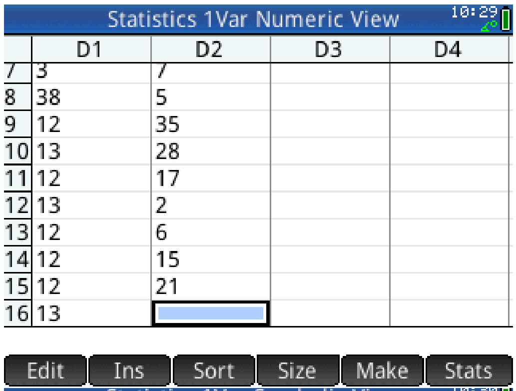
… and on Saturday are as follows:

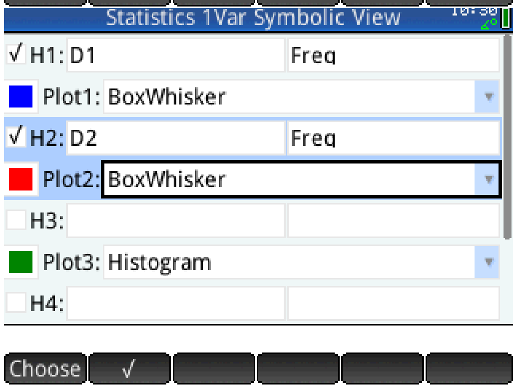
{3, 56, 23, 12, 14, 15, 7, 5, 35, 28, 17, 2, 6, 15, 21} enter this under D2

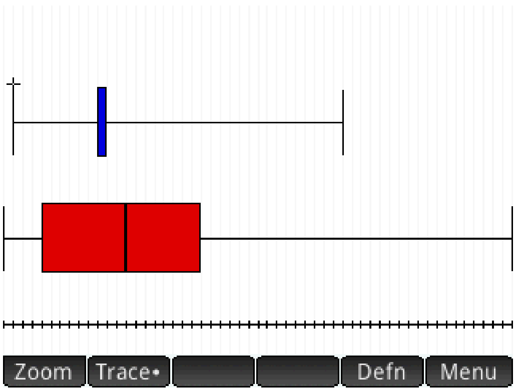
Press Symb and change Plot 1 and Plot 2 from Histogram to BoxWhisker (notice the wide range of other plot types … experiment to see what they are like)

Also, enter D2 for the list associated with plot H2.

Press Plot (then Menu/Zoom/Autoscale) to compare the two distributions.

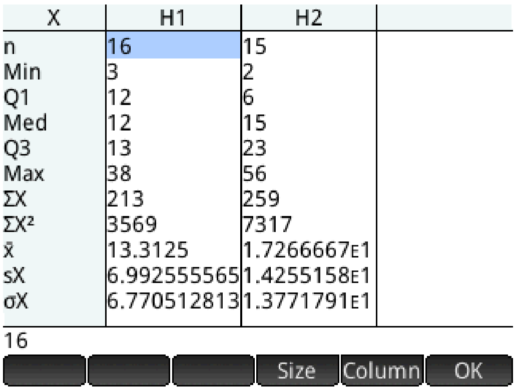






Press Num and Stats to see the summary statistics.

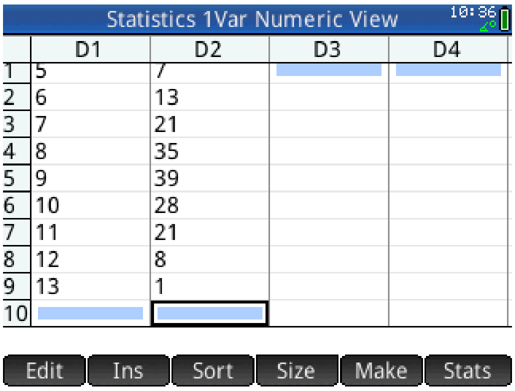
Click OK to exit the STATS view.

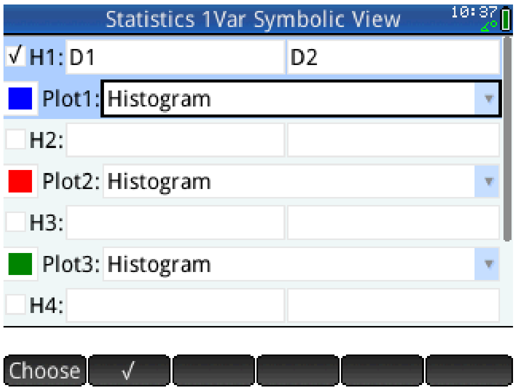


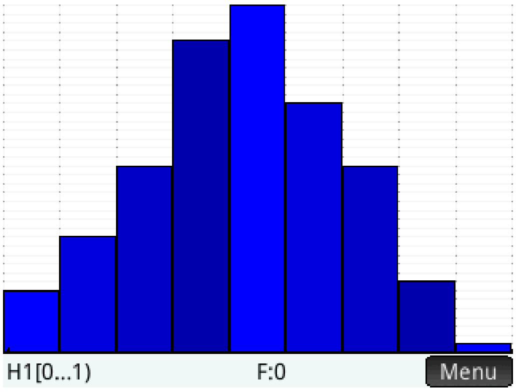
For grouped data use one list e.g. D1 for the values and a second list for the frequencies e.g. D2.

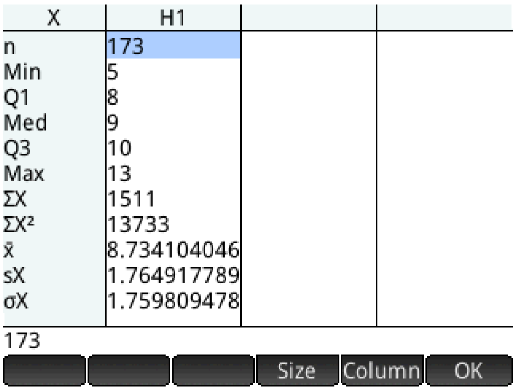
For example, here is a distribution showing the sizes of mens' shoes:

Size	5	6	7	8	9	10	11	12	13
Frequency	7	13	21	35	39	28	21	8	1









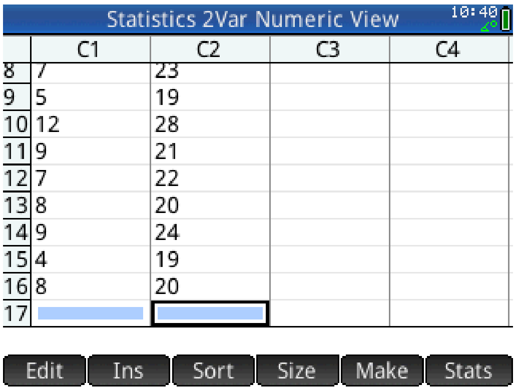
Two Variable Data

Start the 2 variable statistics applet: Apps/Statistics 2Var/RESET/OK/START

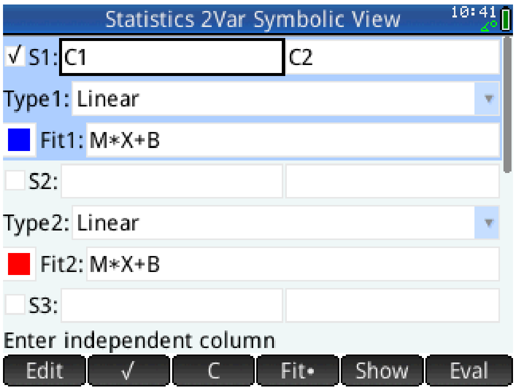
Enter some paired data, e.g. data comparing shoe size with handspan:

Shoe Size	6	9	11	9	10	12	9	7	5	12	9	7	8	9	4	8
Hand Span (cm)	18	23	25	22	26	27	24	23	19	28	21	22	20	24	19	20

Enter Shoe size as C1 and Handspan as C2

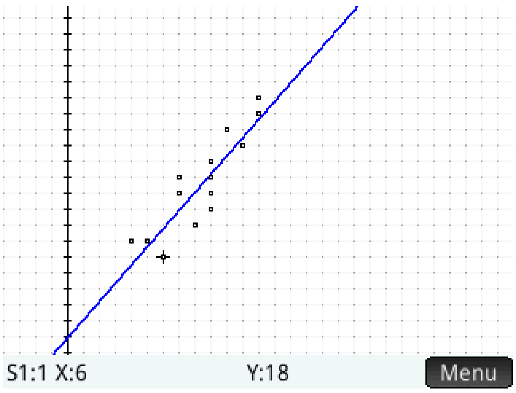


Press Symb and notice that the defualt is to plot C1 against C2 which is what we want.



Press Plot/Menu/Zoom/Autoscale to see a scatter plot with the linear regression line, or simply drag the points into view.

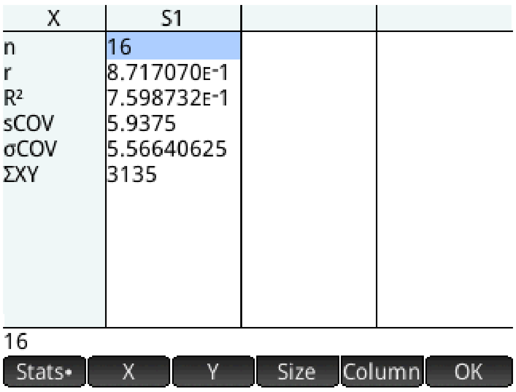
Pressing Fit on the Symb screen will turn the line on and off.



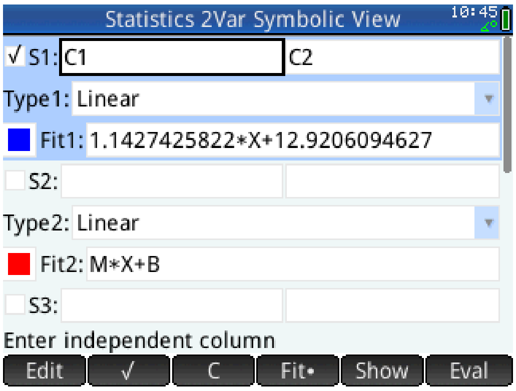
Press Num and Stats to see the summary statistics with the correlation coefficient and covarience.

Notably, the correlation coefficient $r = 0.87$ (t 2 d.p.) shows a reasonably strong relationship.

Press OK to leave the stats screen.



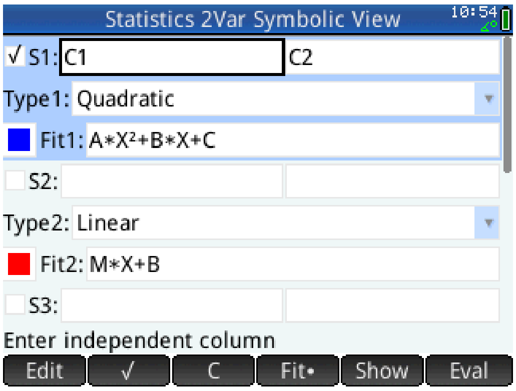
Press Symb Setup again and notice that the details for the regression line are now shown.



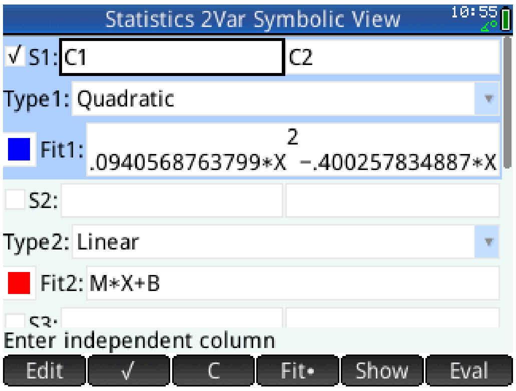
The line is $f(x) = 1.14x + 12.9$ (to 3 s.f.) and our x values (i.e. the first list) showed shoe size. This suggests a model of the relationship between show size and handspan as:

$H = 1.14S + 12.9$ where H =handpsan and S = shoesize.

If we want to see if we would get a better fit with a different model, then we can change the model with the Type options. Navigate to the Type1 entry and choose Quadratic.

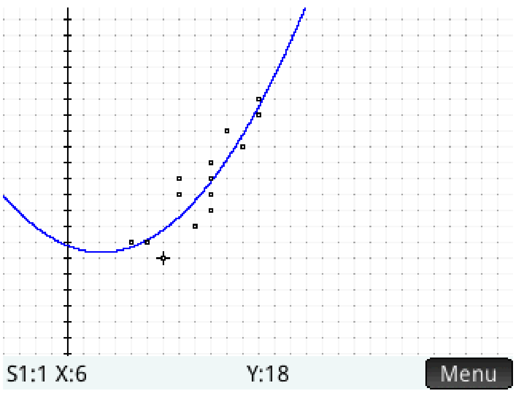


Press Num and Stats to calculate the statistics (many of which are undefined in this new model) then press OK and press Symb again to see the coefficients for the new model.



This gives us a new model of $H = 0.94S^2 - 0.4S + 18.8$ where H =handpsan and S = shoesize.

Press Plot to see the new model.

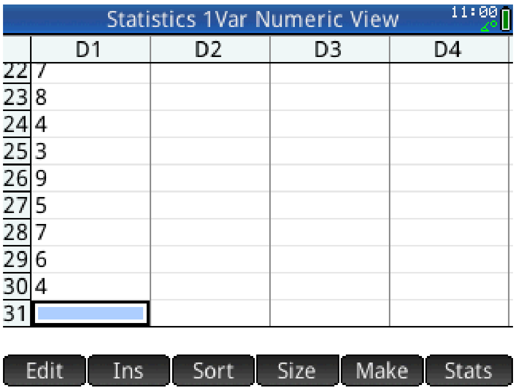


Confidence Intervals

We have some data about the shoe sizes of thirty female maths students. We want to use this data to answer the question, “What is the mean shoe size for UK female maths students?”

Here we are using a sample to ‘infer’ something about the whole population. This is called inferential statistics. The issue will be how confident we can be in the outcomes. We can say that with a given level of probability, the mean will lie within a certain interval. At a 95% level of confidence, we can calculate the range within which the mean will lie, with a probability of 0.95.

To do this, we need to know the mean and standard deviation and the number of data values of the sample.

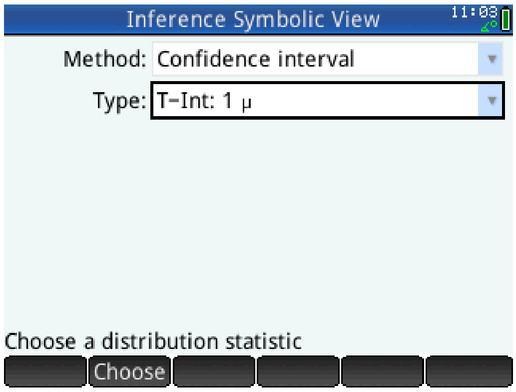


First, we enter the sample data into the 1 variable statistics applet. Press Apps Info, choose Statistics 1Var then Reset/OK/Start then enter the data into list D1

The data is: {2, 7, 6, 4, 5, 3, 4, 5, 6, 7, 4, 3, 5, 4, 4, 3, 5, 6, 6, 8, 6, 7, 8, 4, 3, 9, 5, 7, 6, 4}

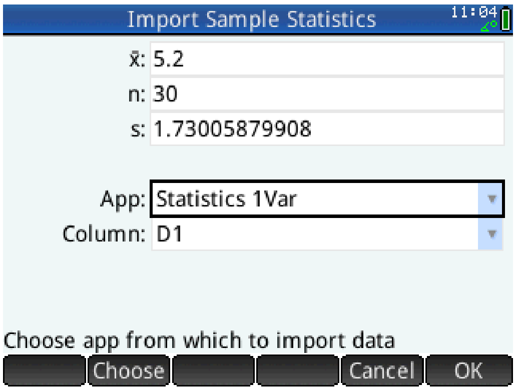
Now start the inference applet:

Press Apps Info, choose Inference then Reset/OK/Start



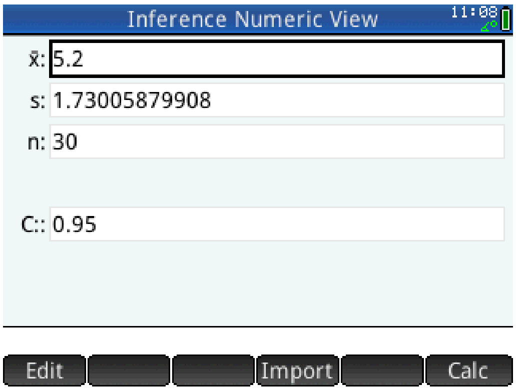
Choose to change the Method to Confidence Interval and the Type to T-Int 1μ

Press Num Setup to get the statistics we entered earlier.



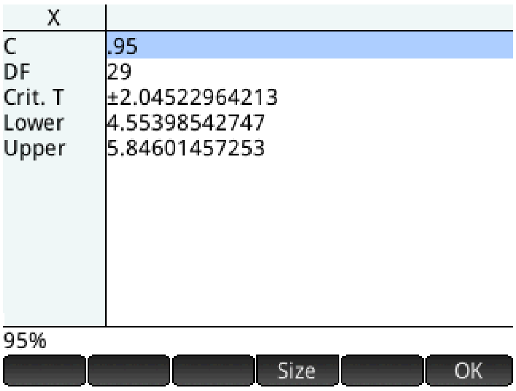
Press Import. Check that we will import list D1 from the Statistics 1Var applet (which it will be expecting) and click OK

You can now see the sample mean $\overline{x}$, the sample standard deviation s and the number of sample data points n. You need to ch ange the confidence interval C to 0.95

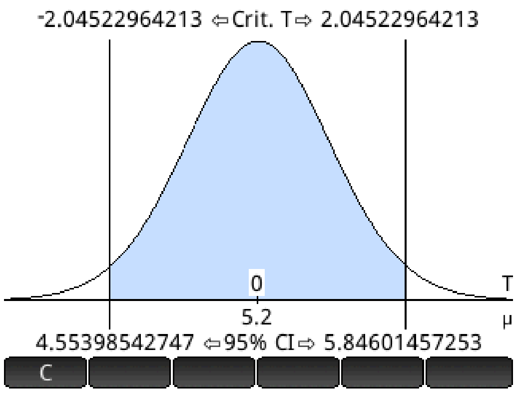


Press Calc to make the calculations.

This tells us that the probability is 0.95 that the shoe sizes of the whole population will fall between 4.55 and 5.85 (to 2 d.p.)



Press  to see this graphically.



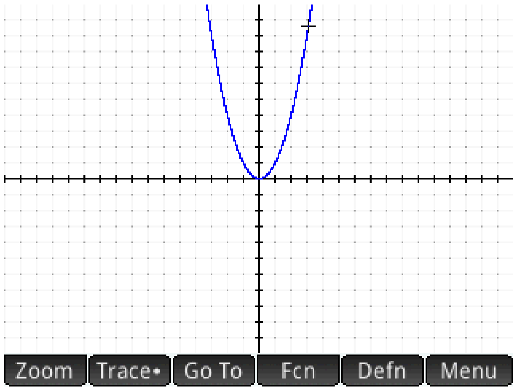
As usual, experiment! See what impact changing the confidence interval has. Look at the other tests available. Research them. See what difference they make.

9. Calculus

It is important to remember that the HP Prime has both numeric and symbolic modes. This section lets you explore how calculus works using the numeric mode.

Differentiation

Let's explore the differential of $f(x) = x^2$. The differential is the function which gives values of the slope of the original function at all values of x.



Open and reset the Function applet:

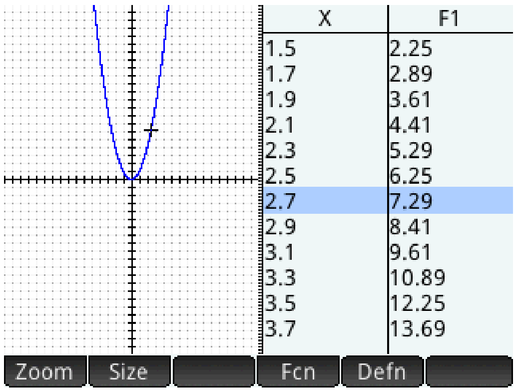
Apps/Function/Reset/OK/Start

Enter the function $f(x) = x^2$

Press Plot Setup to see the graph.

Make sure that Trace is on (the little dot is showing next to Trace) and move the cursor to the furthest right hand/positive end of the visible graph. Now press Fcn and choose Slope.

Before you move the cursor, guess what the slope values will be as you move the cursor to the left hand/negative end of the visible graph. When you have formed a good idea, move the cursor and see.

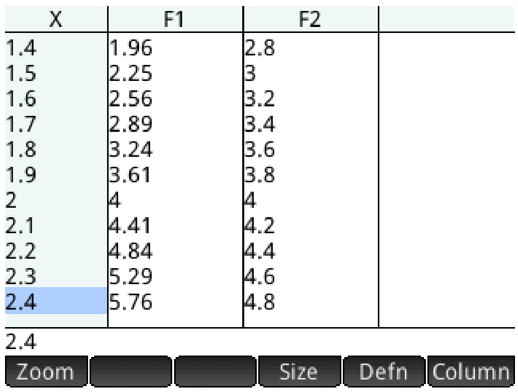


X	F1
1.5	2.25
1.7	2.89
1.9	3.61
2.1	4.41
2.3	5.29
2.5	6.25
2.7	7.29
2.9	8.41
3.1	9.61
3.3	10.89
3.5	12.25
3.7	13.69

Look to see the relationship between the value of the slope and the value of x. (It is easiest to see at key points like the one shown where the slope is 4 and you can see the value of x is 2).

You can see the relationship more clearly by splitting the screen. Press View Copy. Choose the second option (Split Screen: Plot table).

Press Fcn and choose slope, then choose another value of x in the table and find the slope.



X	F1	F2
1.4	1.96	2.8
1.5	2.25	3
1.6	2.56	3.2
1.7	2.89	3.4
1.8	3.24	3.6
1.9	3.61	3.8
2	4	4
2.1	4.41	4.2
2.2	4.84	4.4
2.3	5.29	4.6
2.4	5.76	4.8

Jot down the pairs of values if x and the corresponding slopes that you have found and decide what function shows the relationship between x and the slope. Go back to the Symb page and enter this function for F2(X). Press Num and check that the values for F2(X) are the same as the ones you jotted down.

Repeat this process with a range of functions. The sine function is very interesting. Make slight changes to the functions and see what difference it makes.

(e.g. change $f(x) = x^2$ to $f(x) = x^2 + 1$)

Get a good feel for the differential functions generated by different types of functions.

Integration

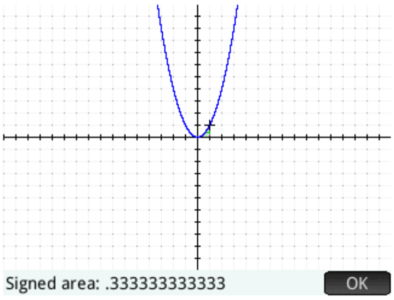
You can apply the same process to get a feel for the integral function. This time we are looking at the area under the graph.

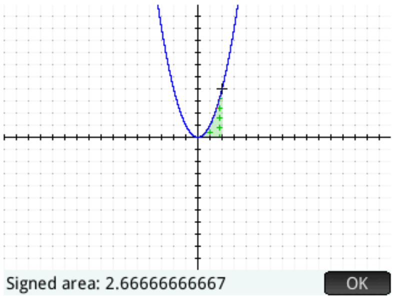
Launch and reset the function applet.

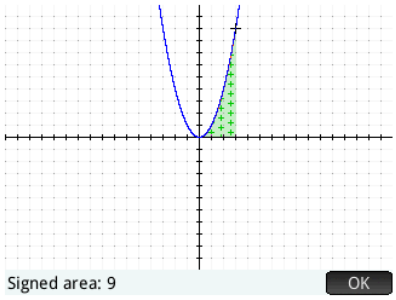
Enter the function $f(x) = x^2$ and Plot it.

Press Menu then Fcn and select Signed area. The cursor will already be at the origin, if not, move it there. Press OK. Now move the cursor to $x = 1$ and click OK.

Now repeat the process starting from $x = 0$ to $x = 2$ … then from $x = 0$ to $x = 3$



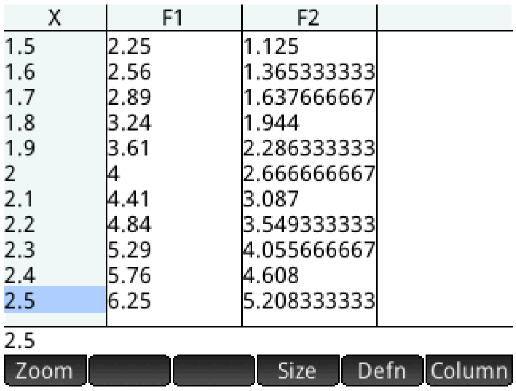




Writing the values as fractions makes things clearer:

x	0	1	2	3
Area from 0 to x	0	$\frac{1}{3}$	$\frac{8}{3}$	$\frac{27}{3}$

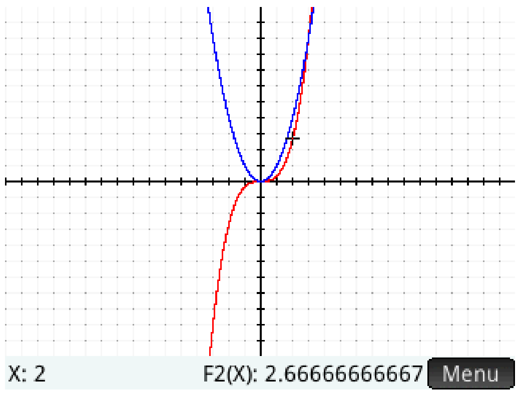
Decide what you think the area function (the integral) is, Press Symb and enter it as F2(X). Press Num and check that this generates the same values.



X	F1	F2
1.5	2.25	1.125
1.6	2.56	1.365333333
1.7	2.89	1.637666667
1.8	3.24	1.944
1.9	3.61	2.286333333
2	4	2.666666667
2.1	4.41	3.087
2.2	4.84	3.549333333
2.3	5.29	4.055666667
2.4	5.76	4.608
2.5	6.25	5.208333333

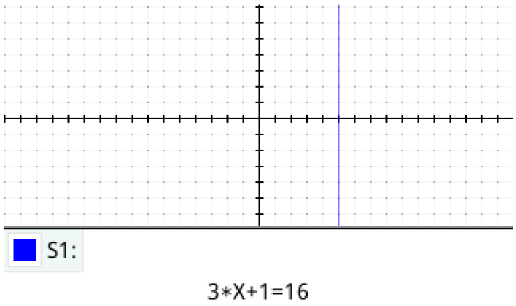
Press Plot and TRACE over F2(X) to check.

Again, do this with different types of functions. Make small changes to existing functions. The aim is to get a feeling for how the area function varies. To be able to work out exactly what the function is from this small amount of data would be very difficult for most functions. But the key point is to experiment and get that all important feeling for how calculus works.



10: Identities. Sometimes True, Always True, Never True

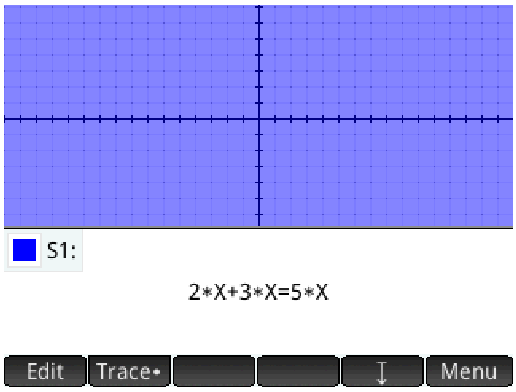
If we look at the statement $3x + 1 = 16$ we can say immediately that this is true only when $x = 5$. Compare that to the statement $2x + 3x = 5x$. That is true for any value of x. It doesn't matter what x is, 2 times a number plus 3 times the same number is always the same as five times that number. Now compare with the statement $\sin(x) = 2$. There is no value of x where $\sin(x) = 2$ because the values of $\sin(x)$ are always between negative and positive one.



So:

- $3x + 1 = 16$ is sometimes true … in fact true in one instance $x = 5$
- $2x + 3x = 5x$ is always true … it is true for all x
- $\sin(x) = 2$ is never true.

See what happens when we type these statements into the Advanced Graphing App in HP Prime.

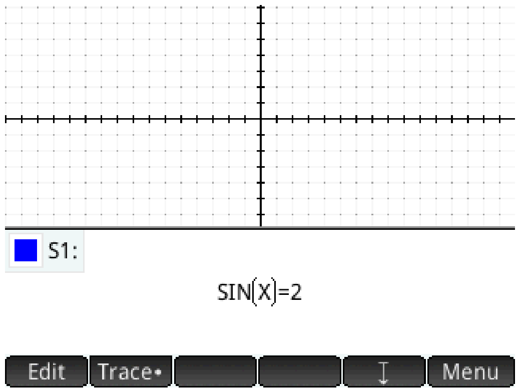


1. Click Apps Info
2. Select Advanced Graphing
3. Type the statement into the line for S1
4. Click Plot Setup
5. Press Menu then Defn (you can drag the graph into a more suitable position if you like)
6. Click Symb Setup to edit the statement, press backspace to delete, then type the next statement and continue as before.

The graph for $3x + 1 = 16$ is a graph of $x = 5$. These two statements are the same. The solution is $x = 5$

The graph of $2x + 3x = 5x$ is everything. It is always true.

The graph of $\sin(x) = 2$ is nothing. It is never true.



Mathematically we are very interested in situations of the first two types. An equation which is always true is called an **identity**. Essentially the statements are identical. Equations which are sometimes true have **solutions** and we are interested to know which equations have solutions and how many they might have.

In both cases we can take the statement and manipulate the algebra to see if we can find statements which are solutions or are identical.

For $3x + 1 = 16$ the solution is $x = 5$

But $2x + 3x = 5x$ is always true so it is an identity. We write it: $2x + 3x \equiv 5x$

Note: Just because the screen shows all, some or none shaded does not prove that these are the only outcomes, just in the range that the screen is showing. You could zoom out (press the – key a few times, which is more convincing, but you cannot see an infinite range!) Nonetheless this gives a very good visual indication.

Activity 1

Test these statements with the Advanced Graphing App. Are they sometimes, always or never true? In each case, if it is sometimes true, what is/are the solution(s)? If it is always true, then the statement is an identity, so re-write it with the identity symbol. If it is never true, explain why.

1. $2\sin(x) = 2$ (remember the calculator is probably set to radians)
2. $x^2 = x + 6$
3. $x^2 = x - 6$
4. $\frac{1}{x+2} = 0$
5. $\frac{1}{x+2} = 1$
6. $5x(x-3) + 2(x+1) = 5x^2 - 13x + 2$
7. $3xy - 2yx = xy$
8. $x^2 - y^2 = (x+y)(x-y)$
9. $x^2 + 6 = y - 5x$
10. $(x-1)^2 = 4 - y^2$

Note: In written algebra when we write xy we mean $x \times y$. However, xy is a perfectly good name for a single variable and so there is a possible confusion. In computer systems you should always type $x \times y$ i.e. X*Y

We hope that you noticed that with one variable (just x), 'sometimes true' gave one or more solutions. However, with both variables involved (x and y) 'sometimes true' gave a relationship. One was a circle, one was a parabola. With one statement on its own we have three types of outcome:

1. An equation with (or without) solutions;
2. An identity;
3. A relationship (most generally called mapping).

Activity 2

Use the advanced graphing App to construct your own examples in each of these three categories.

Equation		Identity	Mapping
With solution(s)	Without solution(s)		

Extend the table as you explore.

11: Parametric Functions: Exploring Projectiles

We can set up graphs with variable coefficients. In this way we can explore the effect of changing the coefficients. A good example of this is to graph a general projectile.

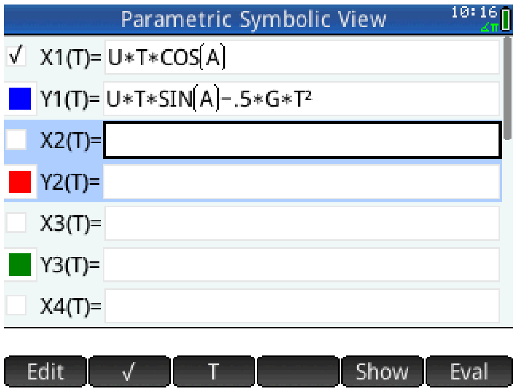
We know the horizontal equation of motion is $x = utcos(A)$

The vertical equation is $y = utsin(A) - 0.5gt^2$

Where u is the initial velocity, A the angle of projection, g acceleration due to gravity and t the time taken.

So, we have two parametric equations with parameter t

First, we enter the equations: (note that T is the parameter so you can use the [$xt\theta n$ Define] key).

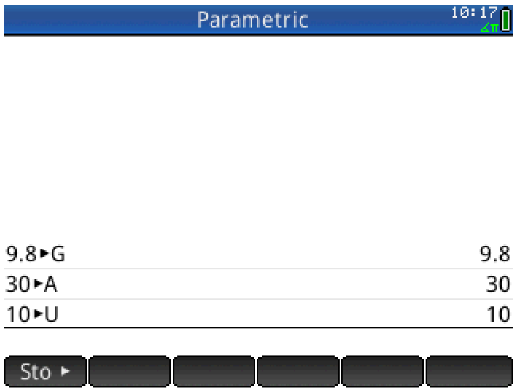


Press [Apps Info], find Parametric, press Reset/OK/Start

For X1(T) type [ALPHA alpha] U × T × COS([ALPHA alpha] A) then OK

For Y1(T) type [ALPHA alpha] U × T × SIN([ALPHA alpha] A) – 0.5 × [ALPHA alpha] G × T [x^2] then OK

If you press Plot now, you will see nothing, since the value of all of the variables defaults to zero. So, we assign values to the variables. First, we will take $g = 9.8$



Press [Settings], type 9.8 press STO [ALPHA alpha] G [Enter ≈]

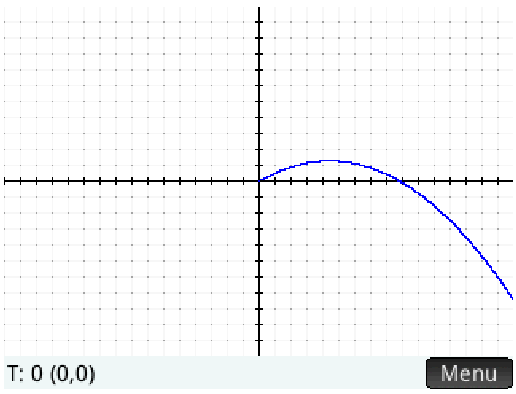
Now choose some sensible values for the other variables, (say):

30 STO [ALPHA alpha] A [Enter ≈]

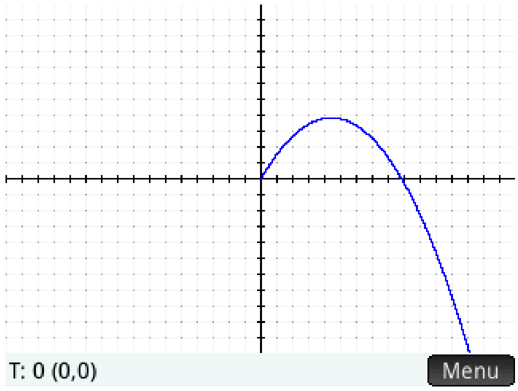
5 STO [ALPHA alpha] U [Enter ≈]

Press [Shift] + [Settings] Settings and set the Angle Measure to degrees.

Now press Plot again.



Use TRACE to find key points in the motion. For example find the time at the highest point and when the projectile lands again. You can pinch and scroll to zoom in on these points.



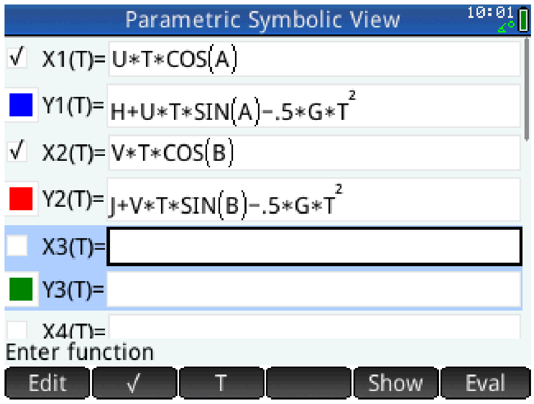
We can go back to the original zoom level using Menu/Zoom/Decimal.

Having set this up you can now explore changing the variables. How much difference does it make if we double the initial angle?

Press [Settings] and 60 STO [ALPHA alpha] A [Enter ≈], then Plot.

Two Projectiles

Now we can add a second projectile to explore when and where projectiles may meet. We can allow for the possibility that either starts at a different height by adding another variable to the vertical equation.

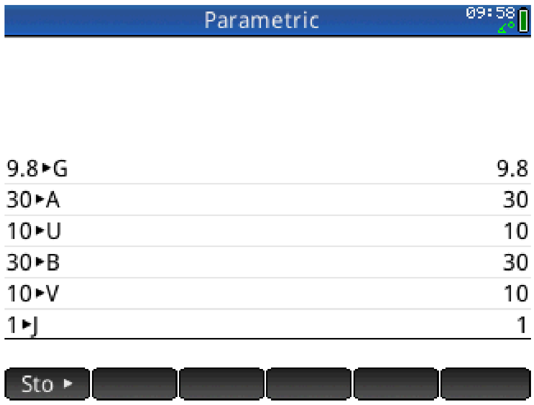


Press [Symb Setup] . Make sure the cursor in the line for X1(T) and press Edit. Use the cursor to move to the beginning of the equation and add H+ to make the equation Y1(T)=H+U*T*SIN(A)-.5*G*T² and press OK

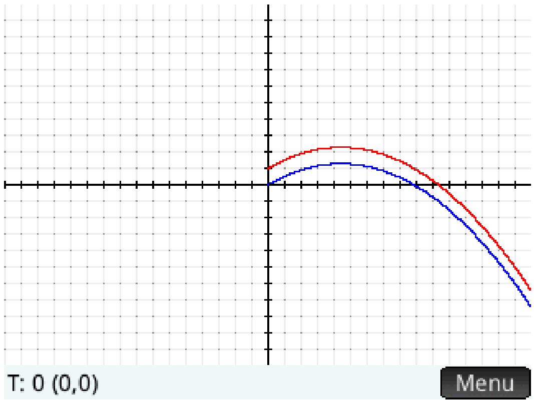
Now enter X2(T)=V*T*COS(B) and Y2(T)=J+V*T*SIN(B)-.5*G*T²

(You can use Copy, Paste and Edit, or you can just type directly).

On the home screen set initial values as before. I would recommend starting with the same values for the second projectile as you have for the first, but setting J=1 which is the initial height of the second projectile.

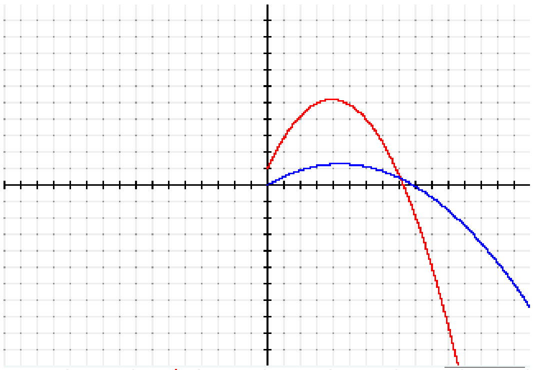


Press Plot to see the paths of the two projectiles.



Now explore:

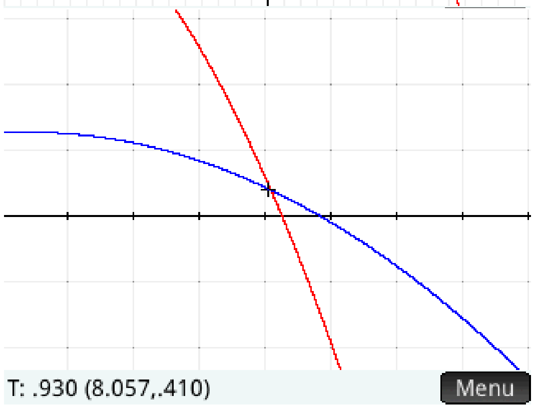
What initial angle for the second projectile is sufficient for it to meet the first projectile before they hit the ground?



(Keep returning to the home screen and trying new values for B using e.g. 40 STO [ALPHA alpha] B [Enter ≈])

Click on the point of intersection to position the cursor, then use the + key to zoom in. Now you can see the position and time of intersection to any level of accuracy you like. (Press Menu if you cannot see the values because the menu items are in the way). Also, use [Shift] + [Settings] and change the Number Format to (e.g.) Fixed 3 to see the values clearly.

Note: If you want to return to the initial screen view: press Menu, then Zoom, scroll down to find Decimal and press that.



Experiment to find the value of B which makes the collision exactly on the ground.

Inclined Planes

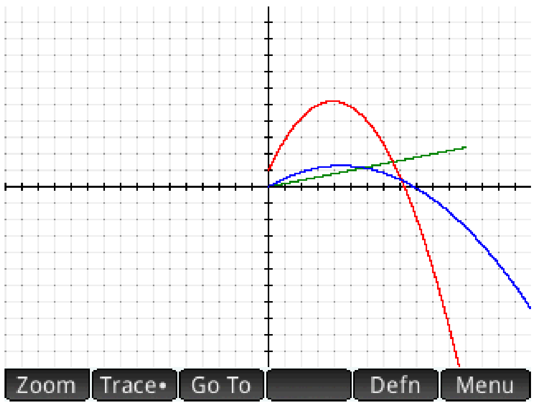
You can also add an inclined plane to your model.

Press [Symb Setup] and enter X3(T)=T and Y3(T)=K*T

In the home screen set a gradient for the plane e.g. K=0.2 using

0.2 STO [ALPHA alpha] K [Enter ≈]

You can now explore the interrelationship between the variables:



Remember:

U and V are the initial speed and A and B the angle of projection of the two projectiles.

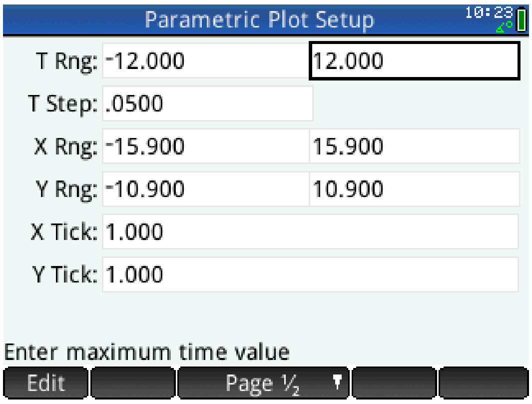
H and J are the initial height above the ground for the two projectiles.

G is acceleration due to gravity for both projectiles.

K is the gradient of the inclined plane.

One final parameter you can explore in your model is time. This is the parameter T which links the two parametric equations, so it is an independent variable. You will have noticed that the inclined plane does not extend to the end of the screen. This is because the domain for T is pre-set, so the calculator does not have to calculate too many values of X and Y. Pressing Shift + Plot Setup gives access to this setting. The range of T (T Rng) is initial set form 0 to 12. We can see what happens if we go back in time by changing the initial setting to -12. (Use the +/- key +/− not the subtract key)

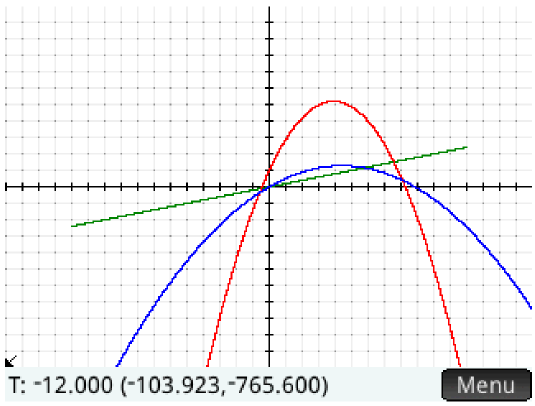
Be careful not to set the range too large or the graph will take some time to draw.



Now press Plot Setup to see the outcome.

You now have a model with two projectiles with variable initial height and a variable inclined plane able to move in positive and negative time.

Create your own problems varying these parameters looking at the time and position of collisions between the projectiles themselves and the projectiles and the ground with varying conditions.



12 Using Your HP Prime in IBO Diploma Exams

It is important to be aware that you are allowed to use your HP Prime in the IBO diploma examinations. So, you should practice using your calculator when you prepare for your exams. There are certain functions which are not allowed in the Exam and your teachers will be responsible for making sure that your calculator complies with the rules. The most important restriction is that CAS functions are not allowed, however, many other facilities need to be turned off as well.

So, when you are getting ready for your exam and practicing with past papers, we would recommend that you set your calculator as it must be in the exam. To do this, you set an exam mode. A VERY important point to note is that if you set a password protected exam mode and forget the password, then your calculator will be permanently set. BEWARE!

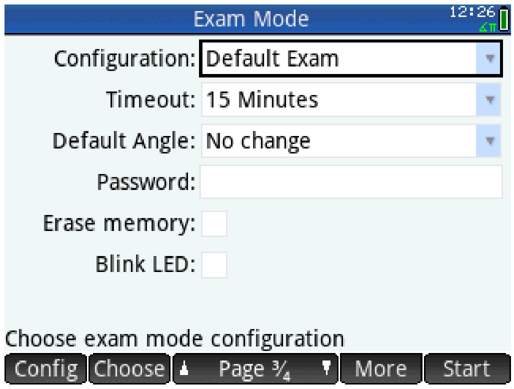
To set the exam mode:

Press Shift + Settings and navigate to page 3.

In the exam the following options must be ticked.

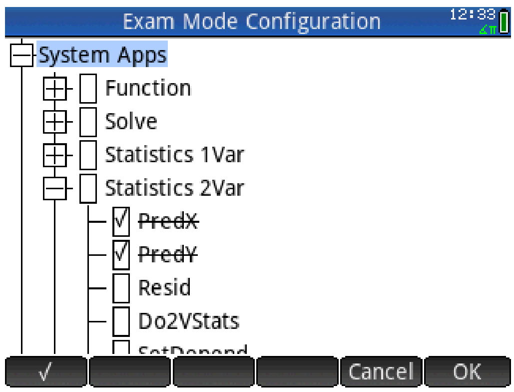
- Erase memory:
- Blink LED:

However, there is no need to do this when you are practicing.



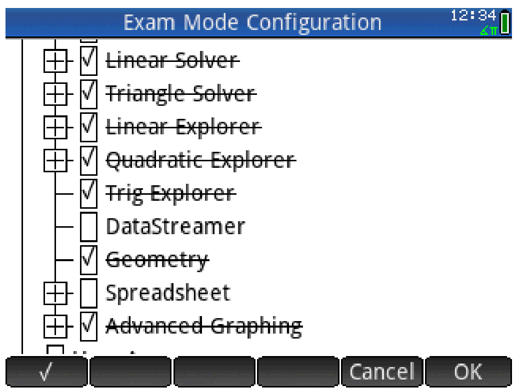
Now press Config and select options to select (and therefore block) as follows.
Press the + next to system Apps to show more options. Then the + next to Statistics 2Var.
Now select:

- PredX
- PredY



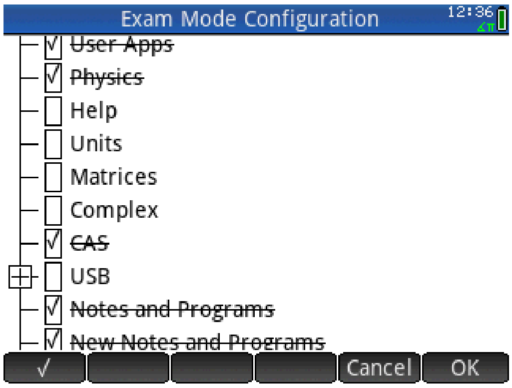
... scroll down to ...

- Linear Solver
- Triangle Solver
- Linear Explorer
- Quadratic Explorer
- Trig Explorer
- Geometry
- Advanced Graphing



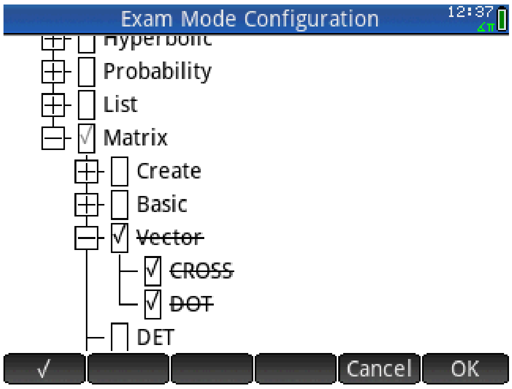
… scroll down to …

- User Apps
- Physics
- CAS
- Notes and Programs
- New Notes and Programs



Scroll down and press the + next to Mathematics. Then the + next to Matrix, then next to vector.

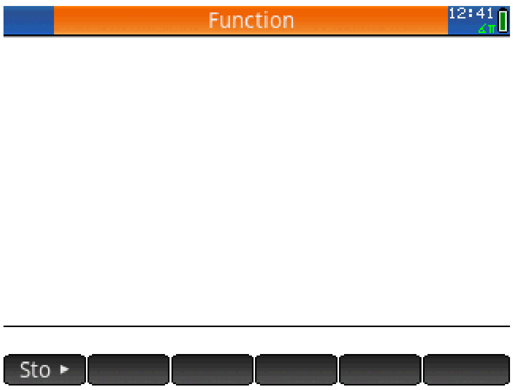
- Vector
  - CROSS
  - DOT



Then press OK.

You can choose how long you want the exam mode to last with the Timeout option. Choose a short period of time. DO NOT set a password. Now press Start and the exam mode will start. You know you are in exam mode because the top of the screen is now orange.

The exam mode will automatically switch off after the time you chose. (You can only turn the exam mode off before this time if you set a password).



HP Prime is an amazingly powerful tool. If you are well practiced and used to using it in exam conditons working with exam questions, then you are sure to find it very helpful. But, do not forget, it will not answer the questions for you, it will simply support you to be an effective mathematician and to free up some thinking space. So, work hard at your maths and you will be successful. We wish you the very best for your International Baccalaureate Diploma with advanced Maths exams and your future work in mathematics.

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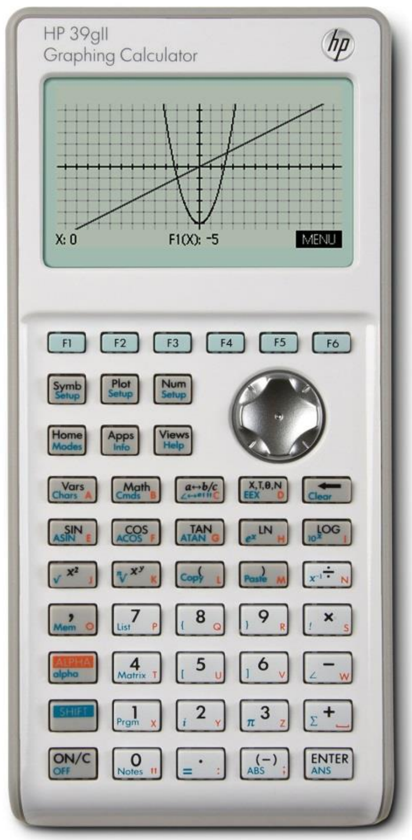
Exploring Maths at Advanced Level

Maths Activities for the HP Prime and HP39gII



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Introduction

This book is aimed at users of the HP Prime and HP39gII graphing calculators, who are studying for an advanced level exam in mathematics (English A Levels, Scottish Highers, French and International Baccalaureat, German Abitur, Netherlands HAVO and VWO 3/4 level and equivalent in other countries). The aim is to provide a range of activities which will help you become a confident user of the calculator while developing your skills in different mathematical topics. There is no attempt to cover the entire course, but there is a good range of topics covered. The power of this technology is in its capacity to generate lots of mathematical information very quickly, so you can get a good feeling for mathematical ideas. Mathematics needs to be explored and with your HP Prime or HP39gII you can get under the skin of the ideas you need to learn about. I hope that you will try out the activities in this booklet and get into the spirit of exploration that you can then use in all of the topics you need to study.

NOTICE: There are significant differences between the HP Prime and the HP39gII. Most important is that the HP Prime has an inbuilt Computer Algebra System (CAS). In this booklet we will not engage with the CAS at all. There will be a separate booklet for activities using CAS. There are also some PRIME only Apps, notably dynamic geometry and spreadsheet which will not be included in this booklet. The remainder of the Apps and the non-CAS operation are broadly similar between HP Prime and HP39gII and this booklet will make use of these areas only.

About the Author

Chris Olley was a secondary school maths teacher in a range of comprehensive schools in London and East Africa. He currently directs the secondary maths PGCE course at King's College London. He has worked with graphing calculators since they first arrived in the late 1980s and has run sessions nationally and internationally on different approaches to dynamic ICT in maths education, of which graphing calculators are an excellent example.

Further Information and Support

Please visit www.hpgraphingcalc.org and join in the discussion threads. Share your ideas and your understanding of mathematics with others.

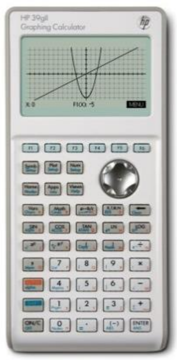
1. Studying Advanced Level Maths with an HP Prime or HP39gII Graphing Calculator

The HP Prime and HP39gII are very powerful calculators. They can find solutions to a vast range of mathematical problems in algebra, calculus, probability and statistics, complex numbers, matrices and much more. Press the Suitcase Key on the HP Prime or the Math key on the HP39gII to see the menu of commands and get a feeling for how much this machine will do.

The big message for advanced level maths students, is that you can use this machine in your exams (those for which a calculator is allowed). I would assume you have bought the calculator because you know this. It is really important that you do not think that the calculator will answer the exam questions for you. You actually have to do the steps in the process. However, the main source of difficulty in an advanced level exam is making small errors along the way. With these calculators you can quickly calculate the solution and check that you have done it correctly and be able to move on confidently. Knowing the answer first is often a very helpful way of deciding what steps to take. Also, being able to see different representations of an object, quickly, helps you decide what the solution could be. With these calculators you can draw graphs and see tables of values of different types of functions, which you can zoom in and out of, make calculations on, even do numeric integration and differentiation.

So, they won't tell you how to solve the problem, but will get you confidently to a solution with a range of better ways of seeing. These are a fantastically powerful tools and all of the evidence from different countries where graphing calculators are used in exams says that if you can use it properly, then it will give you an advantage.

However, it's not just in the exam that the calculator is useful. While you are learning maths, it is vital that you get a good intuitive feeling about how maths works. These calculators are fantastic tools for exploring mathematics. What happens to the graph of a quadratic $y = ax^2 + bx + c$ when you change the coefficients a, b and c? You would need to draw dozens of graphs to get a good feeling for what goes on. With these calculators you can draw as many as you like, changing the coefficients selectively. Working this way helps you get out of the other big problem with advanced level exams. Memorising methods is fine until you find a question you can't recall the method for. That normally means most of them, because they never come up just like you were expecting. Far better to understand what is going on and be able to see the mathematics from different starting points, then you can work your way through even when you can't decide which method to use. Maths is the method and you make yourself a mathematician, by exploring. The HP Prime and HP39gII are the tools to make that possible.



This book has a range of examples. Many of them will suggest areas of maths you can explore. But remember, they are just examples. Use the ideas to explore any new area of maths you are learning. By the time of your exam you will be so skilled in using the calculator that it will be able to support you quickly and powerfully in the exam itself.

2. Basic Operation

The Home Screen and the Suitcase (Prime) or Math (HP39gII) Menu

When you switch the calculator on, you are in the Home screen. Here you can do any type of calculation. Not just arithmetic, but using matrices, summing sequences, calculus, complex numbers and so on. Pressing the Suitcase (Prime) or Math (39gII) key shows you the menu of all of the things you can do on the Home screen. Notice that the Math menu has several sub menus, so make sure that the MATH submenu is selected (by touching Math (Prime) or pressing F1 (39gII)). Navigate with touch (Prime) or the cursor. For details of syntax press the Help key (Prime) or note that the bottom of the menu shows you how to enter things when more than one input is needed (39gII).

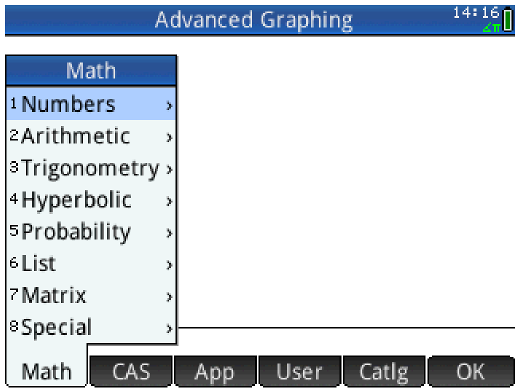


Figure 1: HP Prime 'Suitcase' Menu

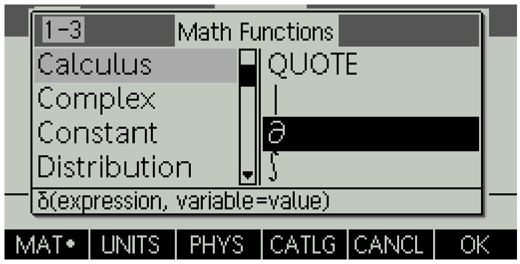
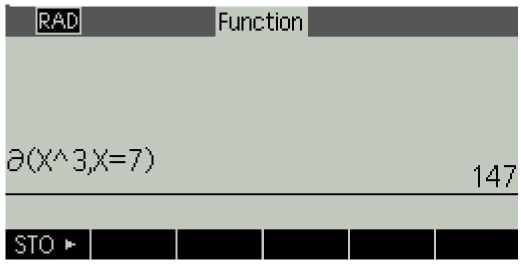


Figure 2: HP39gII Math Menu

Direct Entry versus Template Entry

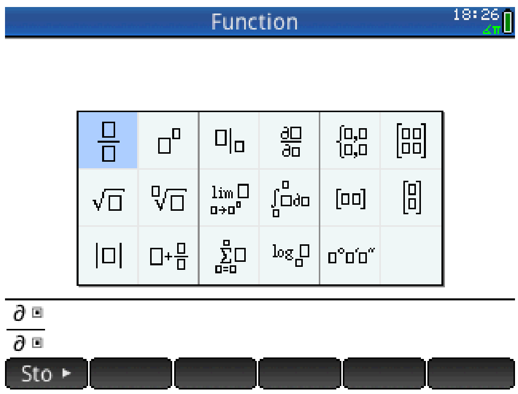
On the HP39gII, to do a numeric differential we find the ∂ command which tells us to enter an expression i.e. the function, then a value, i.e. the value at which we are calculating the differential, i.e. the slope.

Let's calculate the differential of x^3 at $x = 7$. We know the differential is $3x^2$, so at $x = 7$ the result should be $3 \times 7^2 = 147$.



You will have used the X,T,θ,N key to enter the X, the x^y key for the power and the = key (shift and decimal point) to set the value.

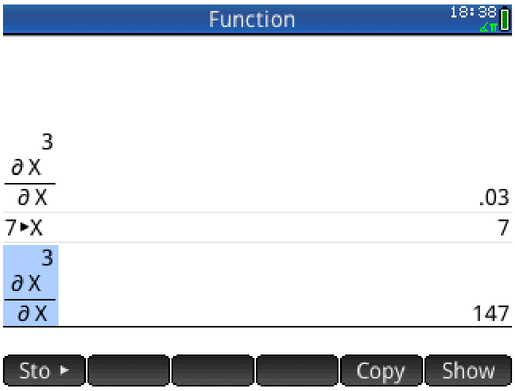
On the HP Prime we have templates for many mathematical functions. Press the template key to the right of the suitcase. Choose the differential. Use the X,T,θ,N and the x^y key to enter the expression then press Enter.



You will get an odd answer here because it will have been evaluated with the whatever value X has.

We wanted X=7, so we must say so. Enter 7 and press the Sto button on the screen and Enter.

Now press on the differential you entered before and Copy (on the screen), then Enter. Now we have evaluated the differential at X=7 (giving the same answer as we got on the HP39gII). The moral here is to enter the values of variable before you use a template.



OK. Now get started. Explore!   Find any functions you already know and make them work for you. Then explore ones you've vaguely heard about.

(If you are doing Physics and/or Chemistry as well, then you will be tempted to check the UNITS and PHYS menus as well. These are on the Math menu on the HP39gII. On the HP Prime they are under Units, the shift option on the template key. I suggest you save this till later, but it is good to see that for scientific work, this calculator will make calculations with correct units attached. It also knows all of the key constants scientists need to know).

Working with Apps

The home screen is where you do calculations. You can always get back to the home screen by pressing the Home button. Pretty much everything else happens through Apps. Press the Apps key and scroll down the list. You will see most of your advanced level topics are covered. All of the Apps work the same way. The **Symb**, **Plot** and **Num** keys show you the three representations of all the maths you can explore with Apps. **Symb** is for the algebraic view, **Plot** shows the graph view and **Num** shows the numeric or table view. Different Apps start in different views according to the maths, for example graphing Apps start in **Symb** mode, so you can enter a function, but Statistics Apps start in **Num** mode, so you can enter the data. The bar at the top of the screen tells you which App is running at any time. If you start up a new App, all of your data from the last App you were using is saved automatically and you can come back to it any time.

You can change the way each of these representations looks using the Setup option on each key (using the SHIFT key). Explore to see what the possibilities are. Finally, you can change the way the screen is organised using the Views menu. For example, you can split the screen and see the graph and table of values at the same time.

There are three main types of Apps:

1. **Function Apps**. Here you enter the data or the function without restriction.

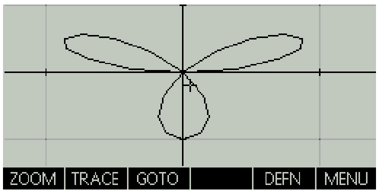
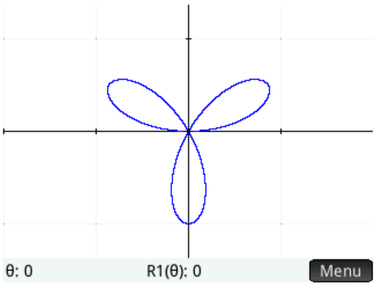


Figure 3: The Polar App



2. **Solver Apps**. Here you get fast solutions to specifics types of problems.

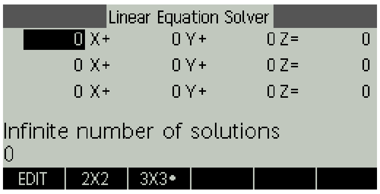
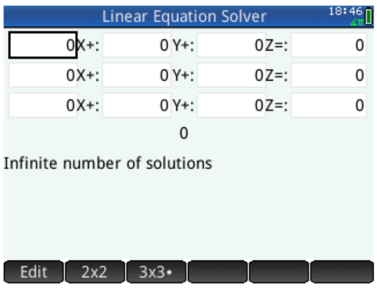


Figure 4: The Linear Solver App



3. **Explorer Apps**. Here you get a pre-configured set up to make exploring specific situation easy. These also contain tests to check

your understanding.

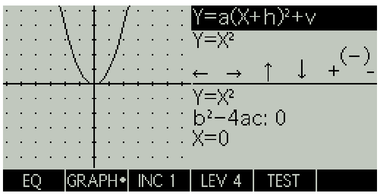
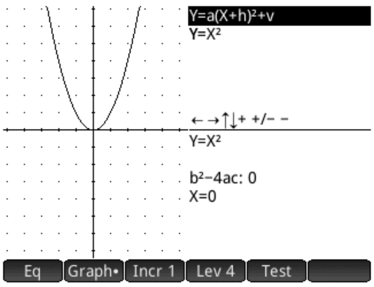


Figure 5: The Quadratic Explorer App



HP calculator users are a great enthusiast community and there are additional Apps created and being created for you to download and add to the functionality of your machine.

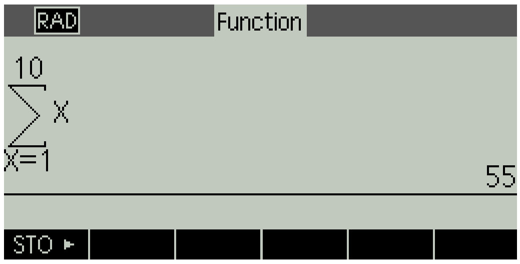
So, check regularly to see what is available, and of course, contribute yourself.

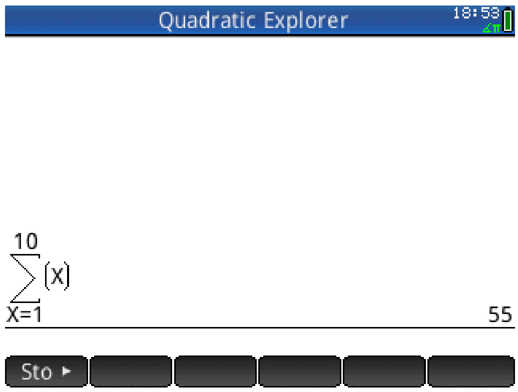
Check at:

- www.hpgraphingcalc.org which is a general purpose support site
- www.calc-bank.com which has programmes and activities

Hints and Tips

- If you are stuck on a menu press ON to cancel menus. The HP Prime has an Esc key which does the same from a more convenient place.
- This calculator has detailed help built in. Press the Help button (Shift Views on the HP39gII) and it will give you help on whatever you were doing.
- On the HP39gII you can show mathematics correctly, like this … Press Modes (SHIFT Home), Press F4 and check the Textbook display option. (HP Prime defaults to this mode)
- You can save apps with your settings preserved and attach notes to remind yourself what you were doing. Press Info (SHIFT Apps) to add the notes. Then select the App in the Apps menu. Click F1 for SAVE and give it a new name. You can then share this with anyone else, or indeed online.
- Install the connectivity software onto your computer. This way, the main operating system of the calculator can be updated when changes become available, you can upload and download Apps and you can type notes for your Apps or Programme your calculator much more efficiently with a keyboard!





3. Graphs and Functions

Linear Functions

It is very important that you are able to look at a function and have a good feel for the size, shape and position of the graph of that function. Your HP Prime or HP39gII are the perfect tools for exploring functions.

In the first instance you should do this for functions of the form $y = f(x)$ using the Function App. Later on you may need to do the same for polar functions, parametric functions and sequence functions, depending on the modules you are taking. There are Apps for all of these, but for now, let's focus on the Function App. Press the Apps key, choose Function, press RESET and OK to confirm, then START.

You should be very familiar with (linear) functions of the form $f(x) = mx + c$. You will know that these are straight line graphs and that they have a gradient of m and they cross the y-axis at $(0, c)$ which is called the y-intercept.

First you should make sure you know the relationship between the gradients of linear functions whose graphs are:

- Parallel
- Perpendicular

Your calculator is ready to receive an input for a function $F1(x)$, so try a simple Linear Function, say $F1(x) = 3x + 1$

Just type 3, press F3 for the X (or press the X,T,θ,N key or press ALPHA X), then +, then 1, then press F6 for OK.

(Notice that the F1 now has a tick next to it, which means that when we choose Plot or Num modes, F1 will show up. We can use F2 for ✓ CHK to turn functions on and off)

Try a function for F2 which will be parallel to F1. The cursor has already moved down to F2, so you can type this directly. Now press the Plot key to see if you are correct.

Press the Symb key, make sure the cursor is on the F2 line and try a new function for F2. Keep trying different functions until you are sure you know the relationship between the gradients.

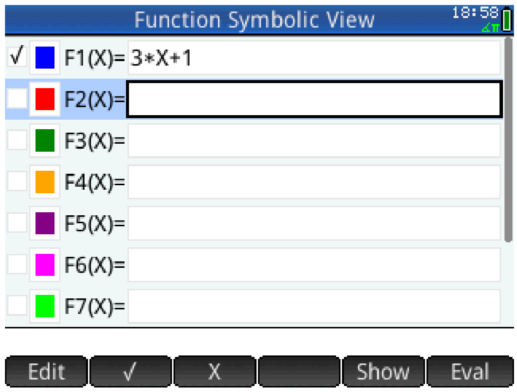
You will need to explore different families of functions:

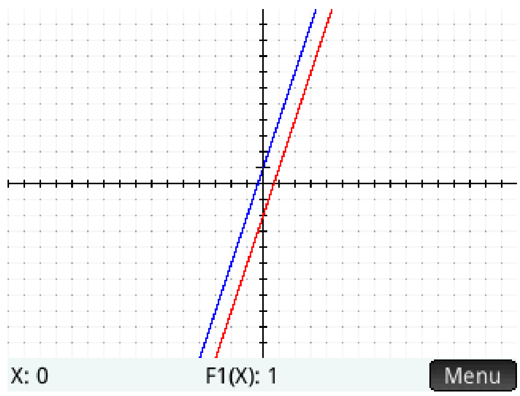
Polynomial functions
$f(x) = ax + b$
$f(x) = ax^2 + bx + c$
$f(x) = ax^3 + bx^2 + cx + d$
(etc.)

Trigonometric functions
$f(x) = \sin(x)$
$f(x) = \cos(x)$
$f(x) = \tan(x)$

Reciprocal Functions
$f(x) = \frac{1}{x}$

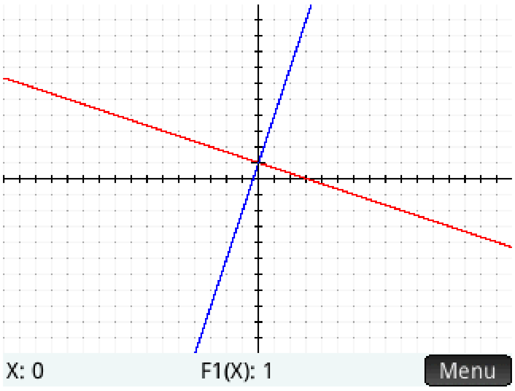
Exponential Functions
$f(x) = a^x$





Now do the same thing for Linear functions whose graphs are perpendicular.

I'm sure you knew the relationships already. But this will have helped you get used to entering and changing functions and looking at their graphs.



Quadratic Functions

Next, you should explore Quadratic functions. There are three standard ways of expressing a quadratic function:

1. Polynomial: $f(x) = ax^2 + bx + c$
2. Factorised: $f(x) = (x + a)(x + b)$
3. Completed square: $f(x) = (ax + b)^2 + c$

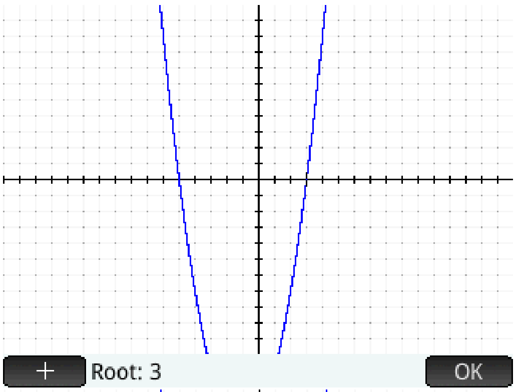
The first one is most familiar, but the factorised form gives the most information quickly (if the quadratic can be factorised). The completed square form probably gives the most complete picture, but the algebra involved in changing to this form is trickier. The important thing is that all three give different insights into the nature of the function. Your calculator will not do the algebra for you, so you will have to practice converting between these three forms with pencil and paper.

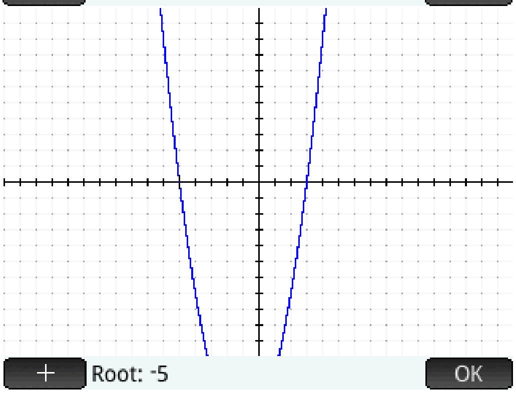
Press Apps, select Function, then RESET/OK/START and enter a function F1 in factorised form, say $F1(x) = (x - 3)(x + 5)$ press OK, then press Plot.

Press Menu, then Fcn. Select Root.

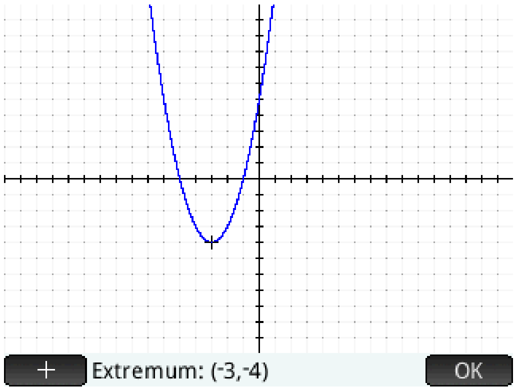
Now move the cursor so it is closer to the second root (clicking left a fair few times will do it). Now press Menu/Fcn/Root again.

The relationship between the roots and the factorised form is pretty clear. Try a few more examples to make sure.





Now get a feel for the completed square form. Replace your F1 with a function in completed square form, say $F1(x) = (x + 3)^2 - 4$ (use the x^2 key to enter the 'squared'). Now look at the position of the minimum point with Menu/Fcn/Extremum
Also, check the positions of the roots as before.

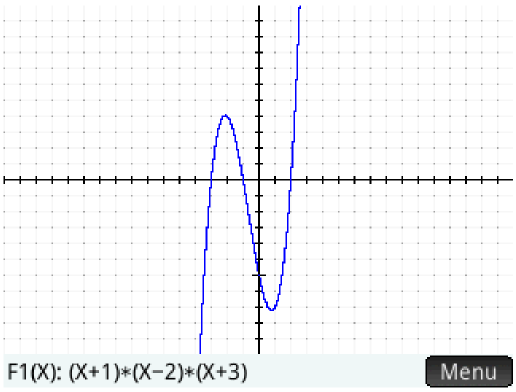


You will need to do some exploration before you can be sure of the relationship between the roots, the extremum and the values of a, b and c in $f(x) = (ax + b)^2 + c$, so keep going until you are quite sure. Then test your theory on new functions in this form.

Now you should be ready to take on the task of exploring the effect of changing a, b and c in the form $f(x) = ax^2 + bx + c$. Work on each one independently. The effect of changing the c is quite straightforward. The effect of changing the a is a bit counter-intuitive. However, just changing the b on its own is quite hard to describe. So, spend some time exploring until you are quite sure you have a good theory that you can describe accurately and simply.

Polynomials

Now you should explore other functions. Firstly, get a feel for cubic functions. Remember that different algebraic forms give different insights into the nature of the function. It is quite hard to imagine the graph of a cubic in the form $f(x) = ax^3 + bx^2 + cx + d$ but much easier in the form $f(x) = (x + a)(x + b)(x + c)$, so factorising first, if possible, is always a good thing.



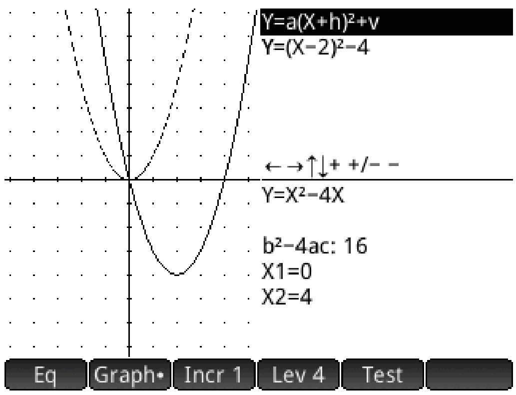
Try some quartic and quintic functions as well, to get a feeling for the differences between different polynomial functions.

Other Functions

You should explore trigonometric, reciprocal and exponential functions in the same way.

When you have a good feel for the general shape and position of these functions, you can explore how the graphs of functions can be manipulated generally.

The best way into this is to have a good play with the graph explorer apps: Quadratic Explorer and Trig Explorer.

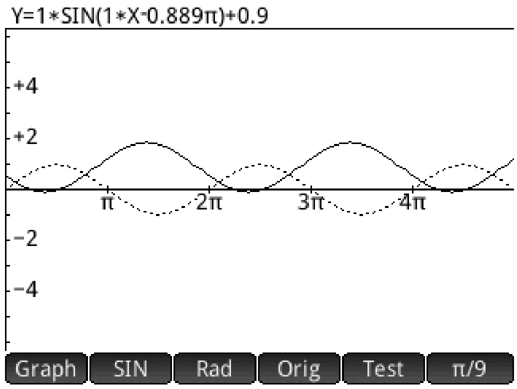


Press Apps and select Quadratic Explorer.
(Best to Reset then Start)

Use the cursor to move the graphs up, down, left and right and spread the graph in and out with the + and – keys. Look at the effect this has on the function.

Work through the 4 levels of difficulty (LEV1 to LEV4) and TEST yourself.

Now do the same with the Trig Explorer applet.



Practice until these conclusions are clear:

A vertical shift of a in the:

+ve direction $f(x)+a$

–ve direction $f(x)-a$

A horizontal shift of a in the:

+ve direction $f(x-a)$

–ve direction $f(x+a)$

A vertical stretch of a :

+ve (increase) a$f(x)$

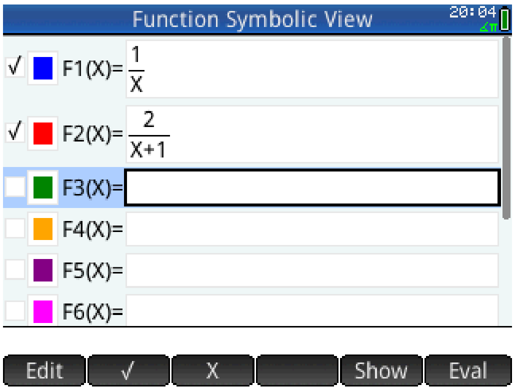
–ve (decrease) $\frac{1}{a}f(x)$

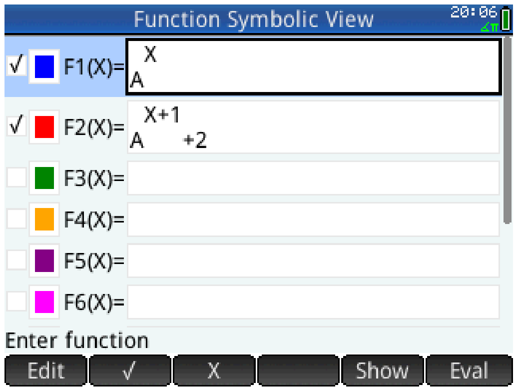
A horizontal stretch of :

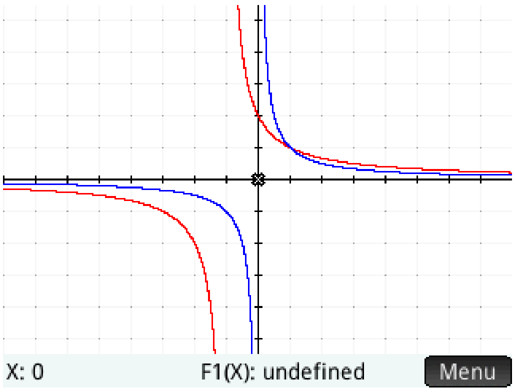
+ve (increase) $f\left(\frac{1}{a}x\right)$

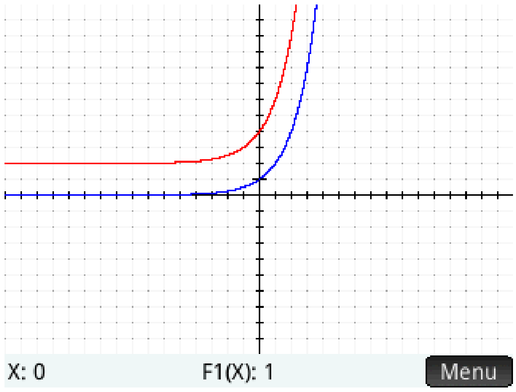
–ve (decrease) $f(ax)$

Now you should explore every different function type. Start with a reciprocal function $f(x) = \frac{1}{x}$ and ensure that you can translate and stretch it horizontally and vertically. Repeat with an exponential function $f(x) = a^x$ and use HOME/Sto/ALPHA/A to change the value of A.









Piecewise functions

We can graph piecewise functions i.e. ones that are defined differently over different part s of the domain.

For example, we can graph a function for which $f(x) = 2x$ when $x < 1$ and $f(x) = 2$ when $x \geq 1$

We write this function as $f(x) = \begin{cases} 2x & x < 1 \\ 2 & x \geq 1 \end{cases}$

Press Apps and launch the Function applet.

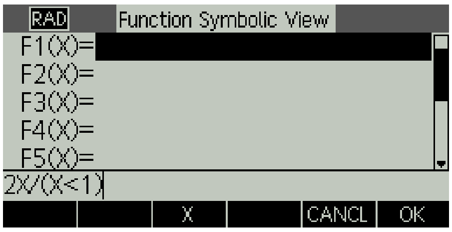
(Reset/OK/Start)

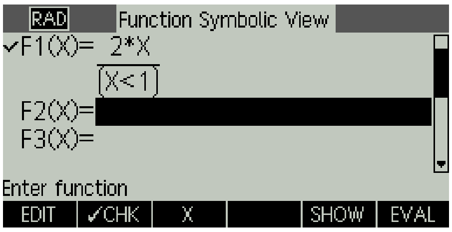
For F1(X) you should

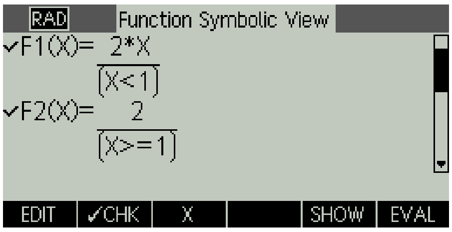
(HP39gII) enter 2*X then press SHIFT Vars (to get the Chars menu) and find the back slash / character. Click OK.

Now enter the domain in brackets (X<2) getting the < from the Chars menu. Click OK to see the function and the domain shown. Now enter the second part of the function as F2(X)=2

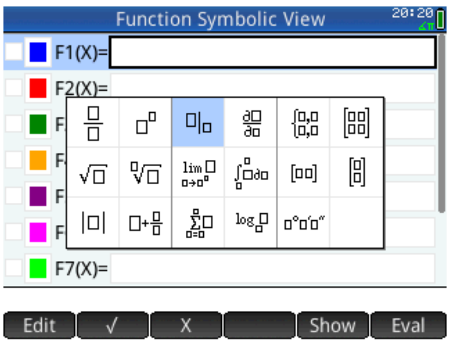
You can find a ≥ symbol but it is easier to find >= which means the same.

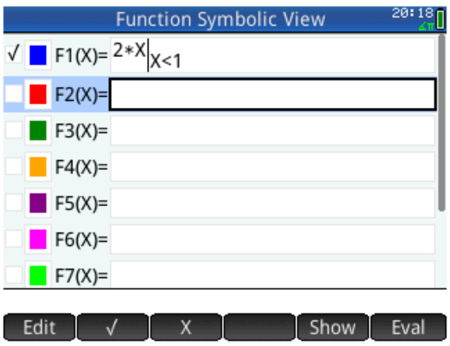


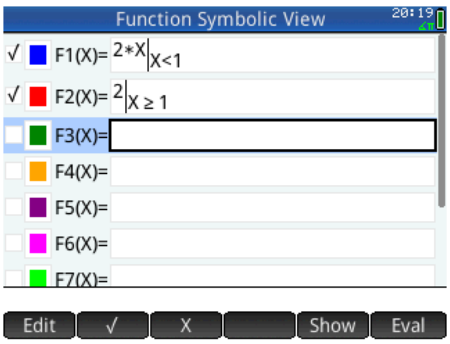




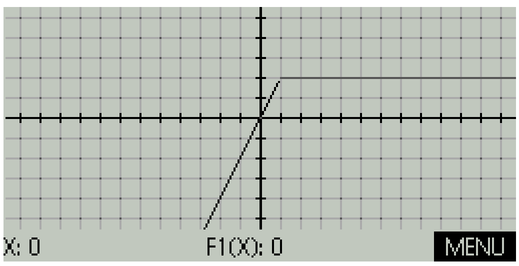
(HP Prime) use the template menu  and enter 2X then the domain X<2 us ing the menu with shift and the 6 key.







Finally press Plot to see the effect. You should explore setting up different piecewise functions, including more than two parts.



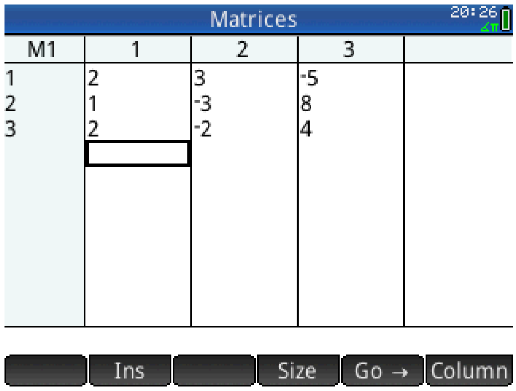
4. Matrices

Mainly you will be using matrices for Linear Algebra. The Matrix button (SHIFT + 4) is the editor. Here you can create Matrices in a convenient way. You can also type matrices directly onto the Home screen. When you have created a matrix, you should store it as a variable like M1, M2 etc.

Let's do a task. Solve the linear system:

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 &= 13 \\ x_1 - 3x_2 + 8x_3 &= -13 \\ 2x_1 - 2x_2 + 4x_3 &= -6 \end{aligned}$$

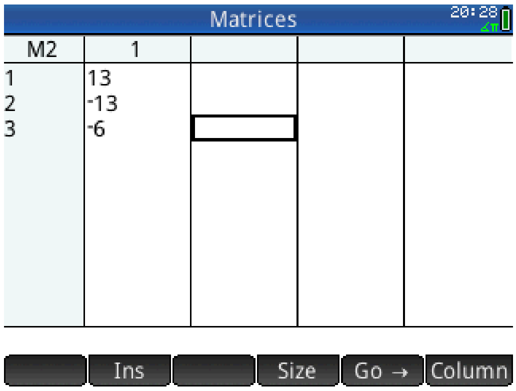
Use the Matrix editor to enter the matrix of coefficients.



This matrix is already called M1 by the editor. Just click Edit to get started and use the cursor keys and OK to enter each value. The Editor quickly works out the size of the matrix. When you are finished, press Home.

To solve the problem we pre-multiply the solution matrix, by the inverse of the coefficients matrix. So create a second matrix M2 with the right hand sides of the equations. This is a column matrix.

Use the matrix editor to enter M2.



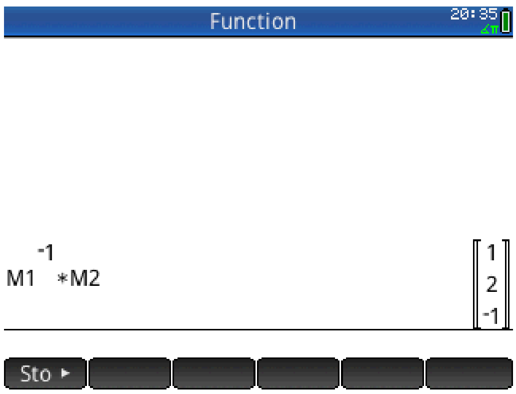
Now we are ready to find the solutions …

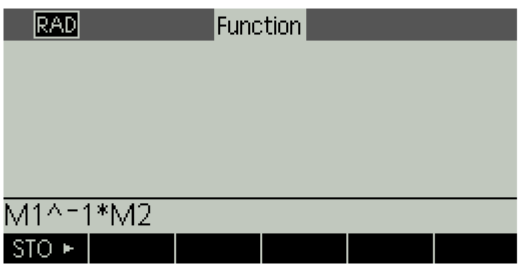
On the Home screen type $\mathrm{M1}^{-1} \times \mathrm{M2}$

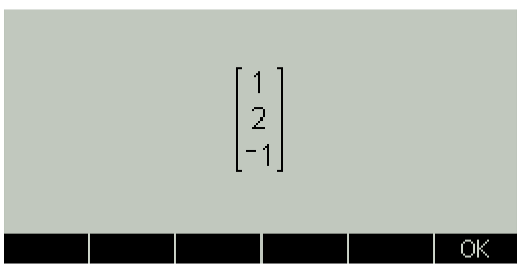
(use the x^y key and type -1)

(HP 39gII) If the Textbook display is turned off, then we can use the Show key (F5) to see the matrix correctly.

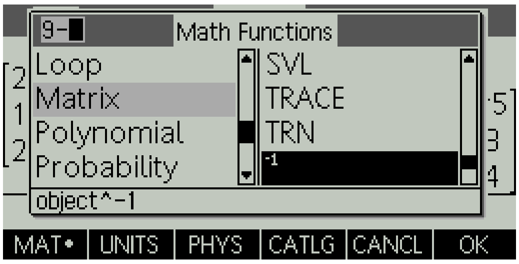
So, we have solutions: $x_1 = 1, x_2 = 2, x_3 = -1$

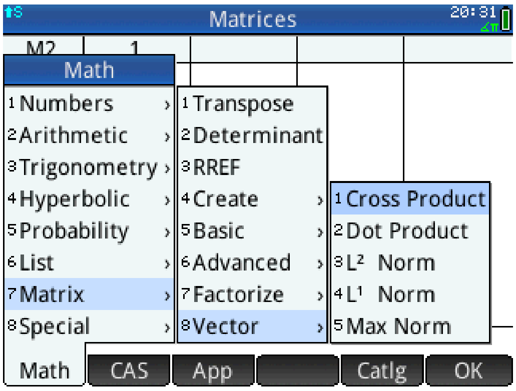




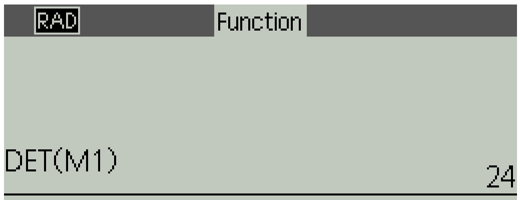


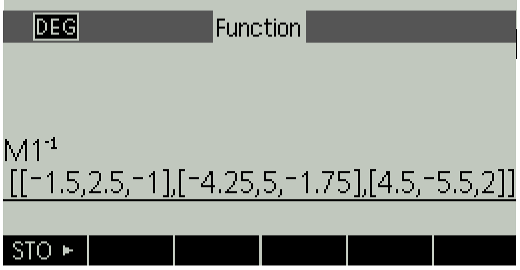
Press the Suitcase key (HP Prime) or the Math key (HP39gII) to find the Matrix section to see the range of operations we can apply to our matrix.





Entering the matrices can be a bit time consuming, but when they are entered you have an enormous amount that you can do with the matrices. So, now explore the other matrix functions available on the Math menu.

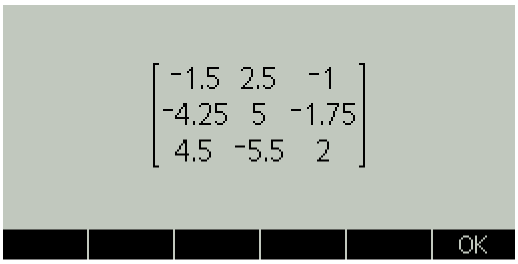




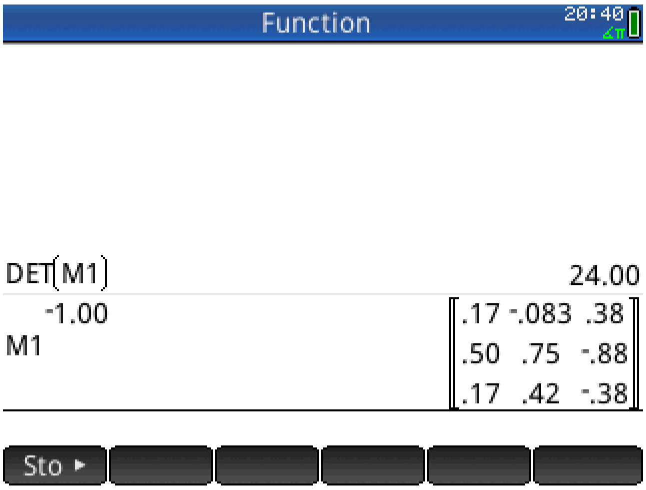
You can find the determinant.

You can find inverse matrices (notice that M2 has no inverse, so will generate a syntax error if you try this).

(HP39gII) Here we have used the up cursor to highlight the matrix and SHOW to see the matrix correctly.



HP Prime shows the matrix in textbook format by default.



You can find Eigen Values and Vectors and do row and column operations. You should experiment to get a feel for all of the available functions and operations.

5. Sequences and Series

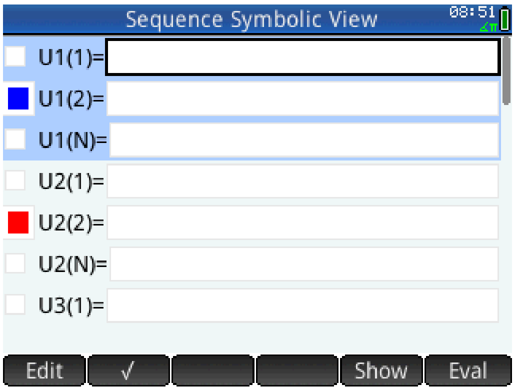
You will be very familiar with different continuous functions. You will have explored the relationship between the function, its graph and tables of values. With your HP39gII you can get a feel for the variation in functions which are not continuous, by exploring sequences and series.

Look at a simple sequence: 3, 5, 7, 9, 11, 13, …

The first term is 3 and the rule is that each number is 2 more than the previous number (we have $u_1 = 3$ and $u_n = u_{n-1} + 2$).

(You may know this as an arithmetic progression with $a = 1$ and $d = 2$).

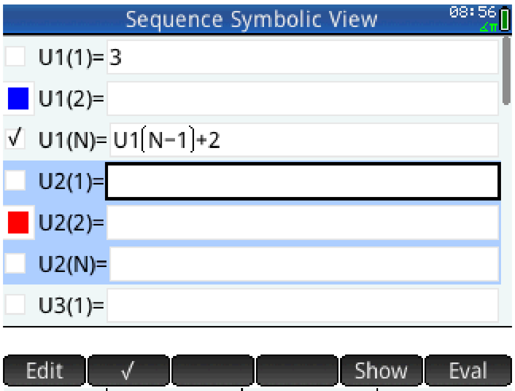
Let's look at this with the sequence app.



Press Apps, select Sequence then Reset/OK/Start

U1 is a sequence. We can enter its first term U1(1), its second term U1(2) and a rule for the nth term. You only need to enter what is needed. The first and second terms are not necessarily needed. In our case, we have a first term and a rule.

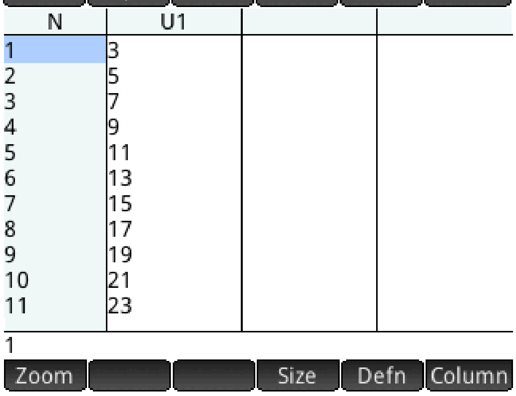
Enter the first term U1(1)=3



Then navigate down to U1(N). Notice that options are added for the (N-1) which we will need.

Use this to enter $u_n = u_{n-1} + 2$ as U1(N)=U1(N-1)+2

Press the Num key to see the sequence.

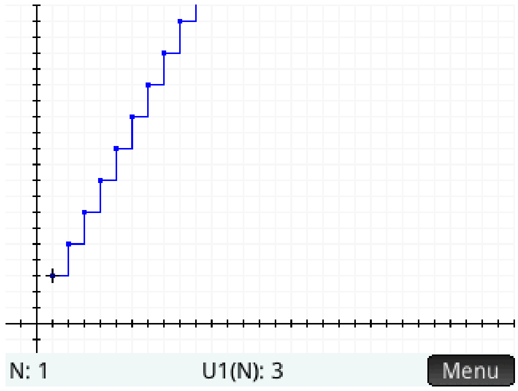


I pressed F4 to turn off the BIG font so you can see more of the sequence.

Use the down arrow to see even more.

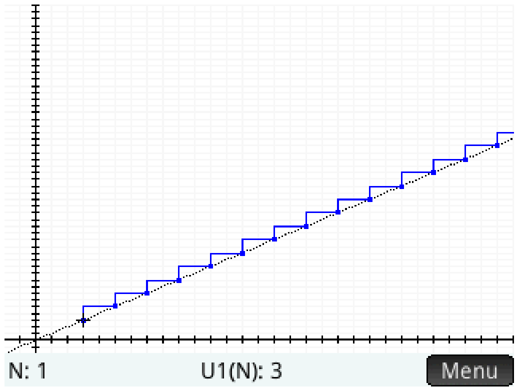
Notice that using the up arrow will not allow us to go back further than the first term, because we entered a value for U1(1) saying that this is where the sequence starts.

Now press Plot to see what the sequence looks like graphically.



Press Shift Plot to see the plot setup options.

I have changed the plot type to Cobweb and increased the end of the Y range (YRNG) to 50 to see more of the sequence.

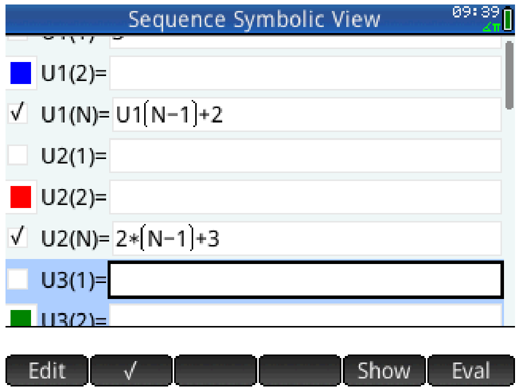


We have specified the sequence with a term-to-term rule. Now we can compare with a position to term rule, i.e. a rule for the nth term.

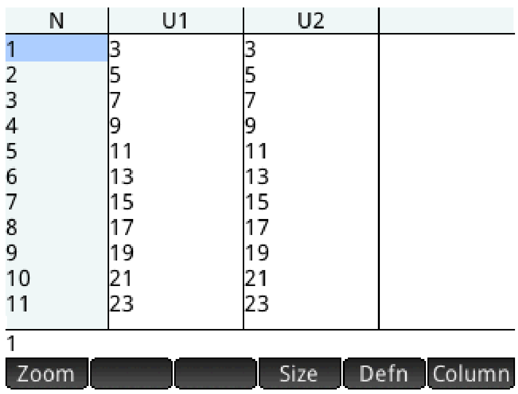
The first term is three and the sequence is increasing by two for each extra position so the position-to-term rule must be $u(n) = 2(n - 1) + 3$. Notice it must be $n - 1$ because the first term has zero 2s added on.

Press Symb and enter this rule as the second sequence U2. Enter the rule as U2(N)=2(N-1)+3

(On the HP39gII notice that when you do this U2(1) and U2(2) are greyed, because with a position-to-term rule, any value can be calculated, so the initial terms are irrelevant).



Press Num and compare the sequences.



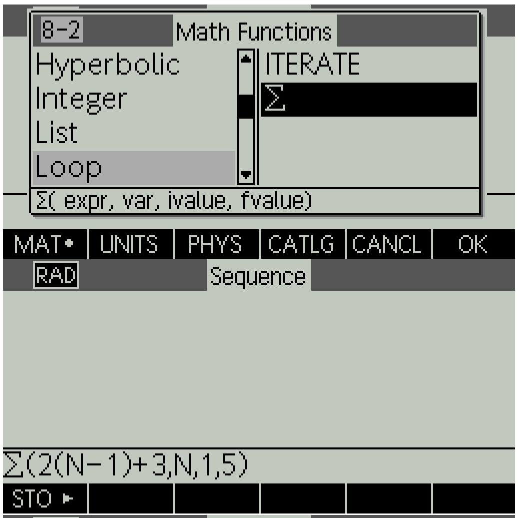
This provides an excellent platform to explore different sequences and practice finding position-to-term rules to match term-to-term rules.

Now that we know the position-to-term rule, we can find values for the corresponding series.

To find 3+5+7+9+11, we are summing terms in the sequence from $n = 1$ to $n = 5$

So, we find $\sum_1^5 2(n-1)+3$

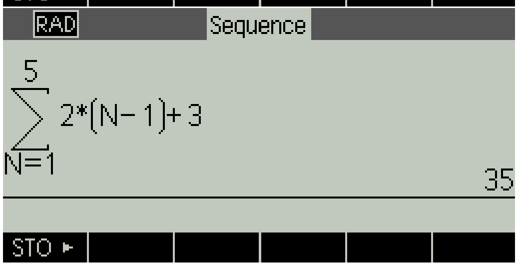
(HP39gII) Press Math, navigate to Loop and choose $\sum$



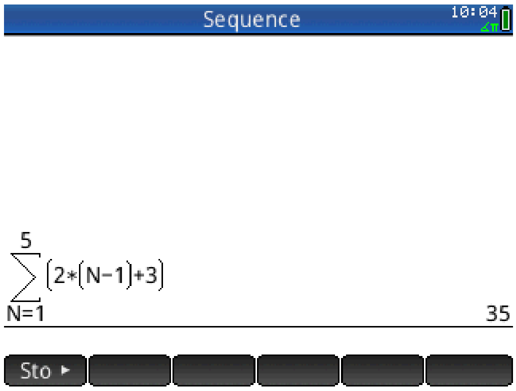
Note that on the menu, it tells you how to enter the values. Click OK, then enter the expression $2(n-1)+3$ using the ALPHA key to enter the N. The *var* is N and the limits are 1 then 5. Make sure to put the commas in between each entry.

Now press ENTER to evaluate.

(This is how it appears with textbook display switched on).



HP Prime) Use the template key



Check that you agree with the result! (3+5+7+9+11 = 35)

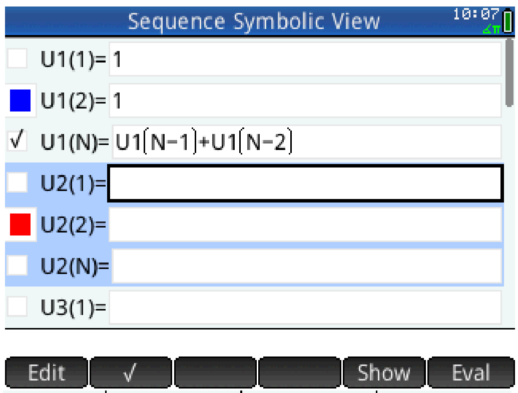
You can easily change the values by using the cursor to select the entry, then pressing Copy and editing. You should explore the effect of making changes to the limits and to the rule.

You can easily explore more complicated sequences. An

excellent example would be Fibonacci sequences.

1, 1, 2, 3, 5, 8, 13, …

Here, we know the first and second terms and a rule that each term is the sum of the preceding two.

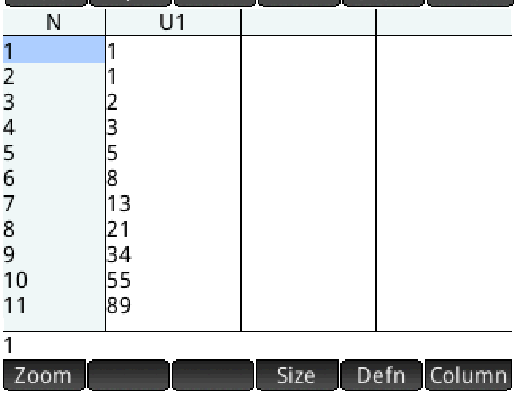


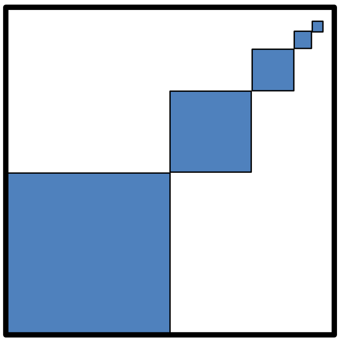
Relaunch the sequence applet and reset it, ready to enter the new sequence.

We have U1(1)=1, U1(2)=1 and U1(N)=U1(N-1)+U1(N-2)

Check that it has worked by looking at the table.

(Press Num)





A square is divided into quarters and one quarter is shaded. A quarter of the opposite unshaded quarter is shaded. A quarter of this quarter is shaded and so on as shown. What fraction of the original square is shaded?

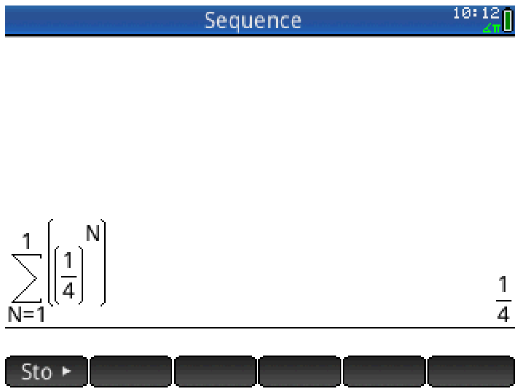
If the original square has unit area, then the shaded part is $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots$

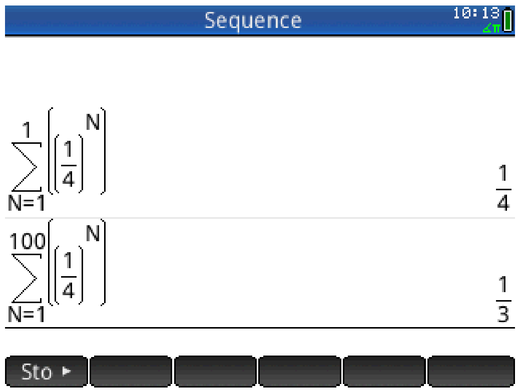
That is, adding up the terms of the sequence with $u_n = \left(\frac{1}{4}\right)^n$

So, see what happens to this value as n increases.

Enter $\sum_1^1 \left(\frac{1}{4}\right)^n$

Press *a*↔*b*/*c* to see the outcome as a fraction.





Use the up arrow to COPY and EDIT the sum. Keep increasing the last term to see how the sum is changing. What value does it tend towards as the number of terms increases?

Try to explain how you know that the shaded area must be this fraction of the whole.

6: Projectiles. Parametric Functions

We can set up graphs with variable coefficients. In this way we can explore the effect of changing the coefficients. A good example of this is to graph a general projectile.

We know the horizontal equation of motion is $x = utcos(A)$

The vertical equation is $y = utsin(A) - 0.5gt^2$

Where u is the initial velocity, A the angle of projection, g acceleration due to gravity and t the time taken.

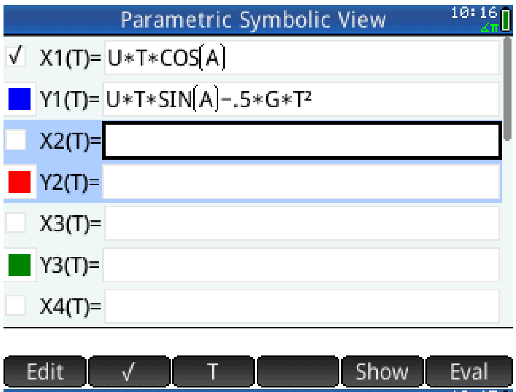
So, we have two parametric equations with parameter t

First, we enter the equations: (note that T is the parameter so use the X,T,θ,N key).

Press Apps, find Parametric, press Reset then OK, then Start

For X1(T) type ALPHA U × T × COS(ALPHA A) then OK

For Y1(T) type ALPHA U × T × SIN(ALPHA A) – 0.5 × ALPHA G × T2 then OK



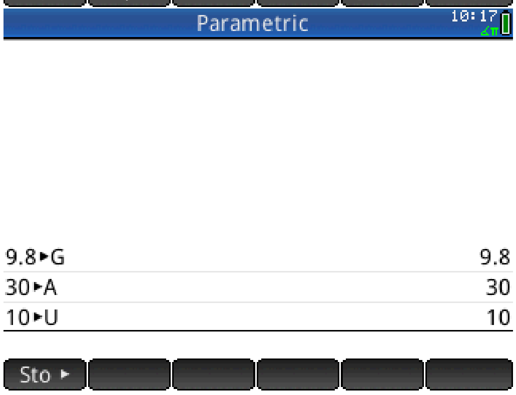
If you press Plot now, you will see nothing, since the value of all of the variables defaults to zero. So, now we assign values to the variables. First, we will take $g = 9.8$

Press HOME, type 9.8 press STO ALPHA G ENTER

Now choose some sensible values for the other variables, (say):

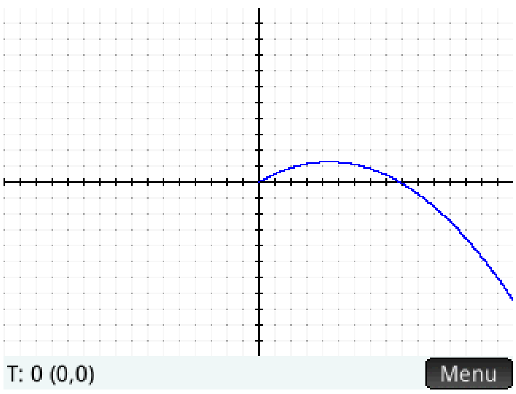
30 STO ALPHA A ENTER

5 STO ALPHA U ENTER

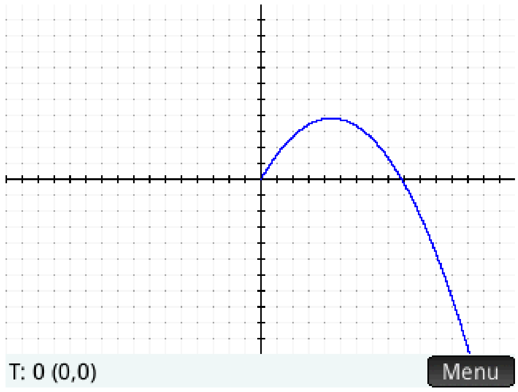


Press (HP Prime) Shift Home Settings or (HP39gII) SHIFT Modes and set the Angle Measure to degrees.

Now press Plot again.



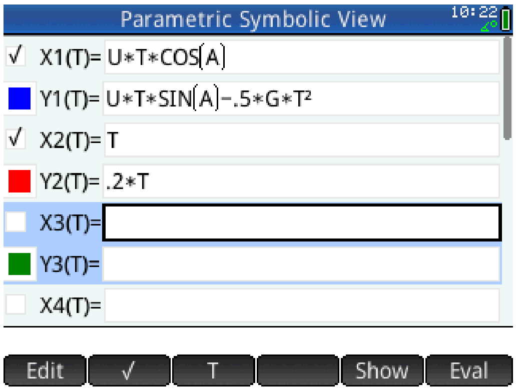
We can use ZOOM Box… to zoom in on the motion and use TRACE to find key points.



We can go back to the original zoom level using ZOOM Decimal.

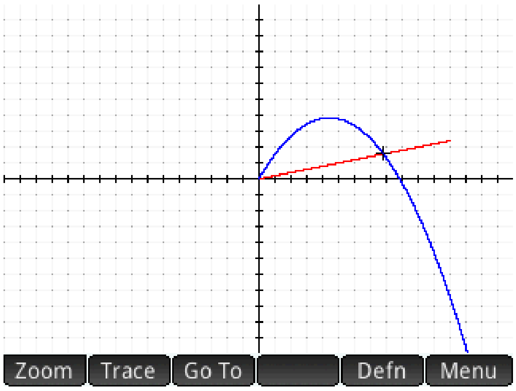
Having set this up you can now explore changing the variables. How much difference does it make if we double the initial angle?

Press Home and 60 STO ALPHA A ENTER, then Plot.



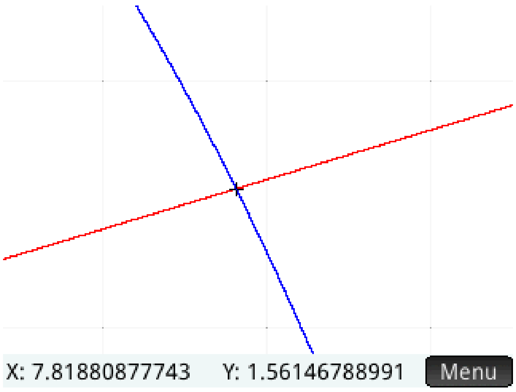
We can solve sloping ground problems. For example, suppose the projectile is launched from a point on a slope with gradient 0.2. We can model this with a function $f(x) = 0.2x$. However, we are in parametric mode, so we use the parameter T for X as $x(t) = t$ and then $y(t) = 0.2t$ so we enter X2(T)=T and Y2(T)=0.2T.

Press Plot to look at the motion.



Now use zoom box to find out when the projectile lands.

First use zoom box to zoom in close on the landing point. Then TRACE to find the value of T. You can increase the precision by changing the T steps using SHIFT Plot. To 2 d.p. the projectile lands after 7.82 seconds.



Finally you can even see what happens going back in time! Press SHIFT plot to Setup the plot. The T range defaults to starting at time T=0, which is logical. But change the TRNG to start in the past e.g. T=-2 …

7. Statistics: Probability Distributions and Inference

For statistical work, you should use the three statistics applets:

- 1 variable statistics (for box plots, histograms etc. with central tendency and spread)
- 2 variable statistics (for scatter plots, correlation and regression)
- Inference (for inferential statistics and confidence intervals)

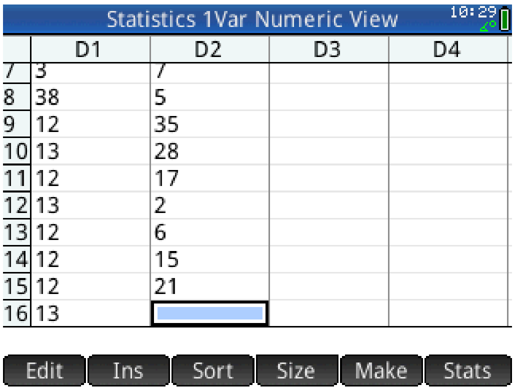
Also, press Math and look at the following sub menus:

- Distribution (binomial, normal, poisson, etc.)
- Probability (permutations, combinations, random numbers, etc.)

One Variable Data

Get started by entering some data for 2 single variable lists.

Apps/Statistics 1Var/Reset/OK/Start

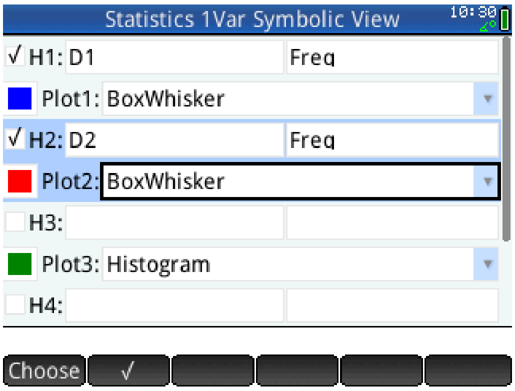


For example, the ages of visitors to a swimming pool on Friday are as follows:

{12, 13, 12, 12, 12, 12, 3, 38, 12, 13, 12, 13, 12, 12, 12, 13}
enter this under D1

… and on Saturday are as follows:

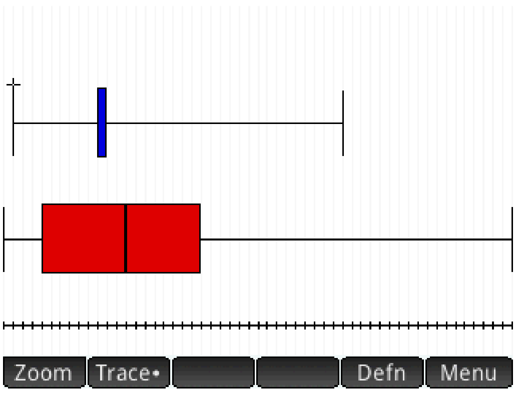
{3, 56, 23, 12, 14, 15, 7, 5, 35, 28, 17, 2, 6, 15, 21}
enter this under D2



Press Symb and change Plot 1 and Plot 2 from Histogram to BoxWhisker (notice the wide range of other plot types … experiment to see what they are like)

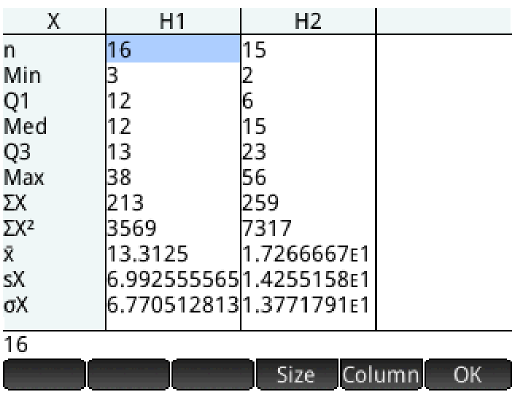
Also, enter D2 for the list associated with plot H2.

Press Plot (then Menu/Zoom/Autoscale) to compare the two distributions.



Press Num and Stats to see the summary statistics. (HP39gII) Press down to see mean and standard deviation(s).

Click OK to exit the STATS view.

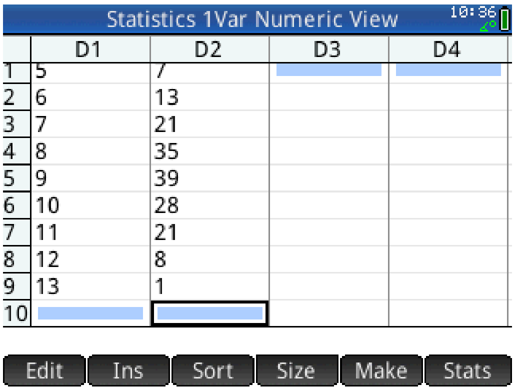


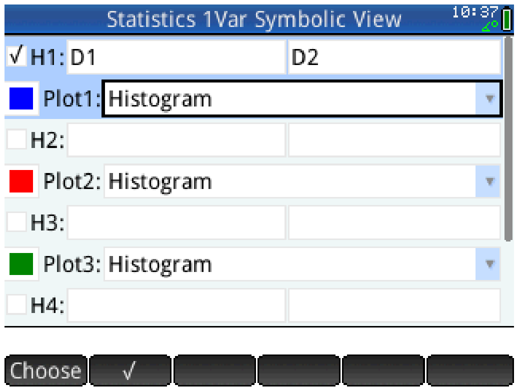
X	H1	H2
n	16	15
Min	3	2
Q1	12	6
Med	12	15
Q3	13	23
Max	38	56
ΣX	213	259
ΣX²	3569	7317
x̄	13.3125	1.7266667E1
sX	6.992555565	1.4255158E1
σX	6.770512813	1.3771791E1

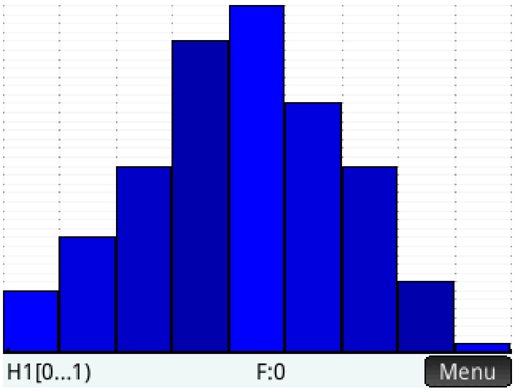
For grouped data use one list e.g. D1 for the values and a second list for the frequencies e.g. D2.

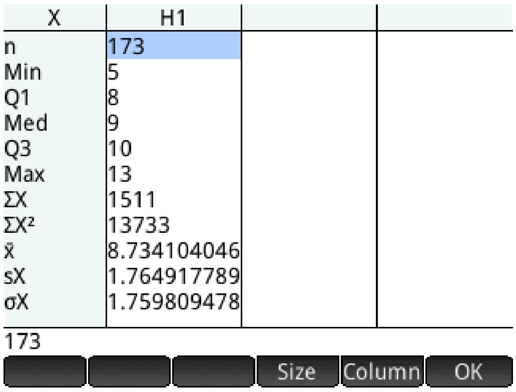
For example, here is a distribution showing the sizes of mens' shoes:

Size	5	6	7	8	9	10	11	12	13
Frequency	7	13	21	35	39	28	21	8	1









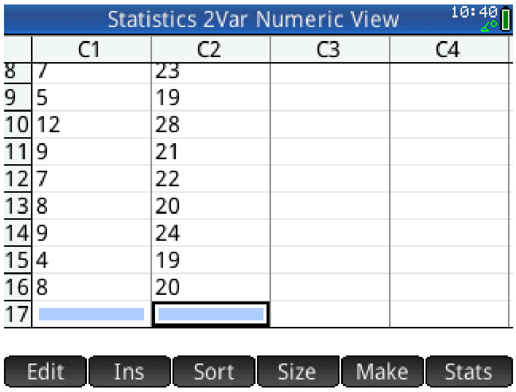
Two Variable Data

Start the 2 variable statistics applet: Apps/Statistics 2Var/RESET/OK/START

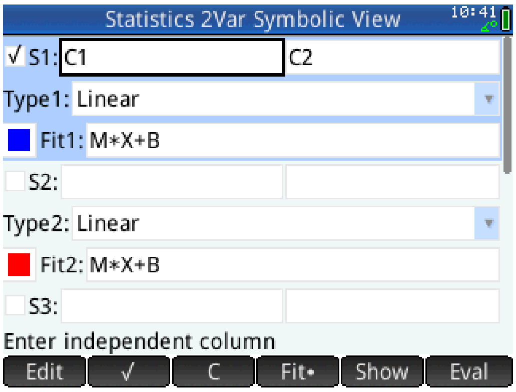
Enter some paired data, e.g. data comparing shoe size with handspan:

Shoe Size	6	9	11	9	10	12	9	7	5	12	9	7	8	9	4	8
Hand Span (cm)	18	23	25	22	26	27	24	23	19	28	21	22	20	24	19	20

Enter Shoe size as C1 and Handspan as C2

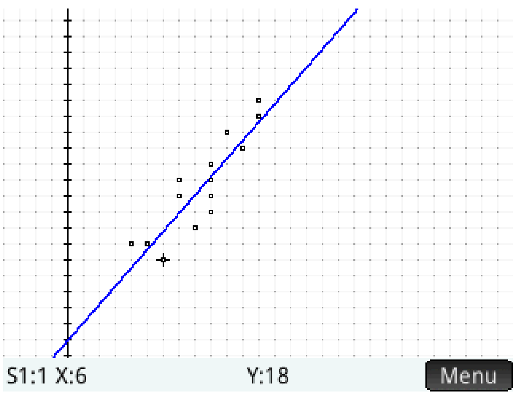


Press Symb and notice that the defualt is to plot C1 against C2 which is what we want.



Press Plot/Menu/Zoom/Autoscale to see a scatter plot with the linear regression line, or on the HP Prime simply drag the points into view.

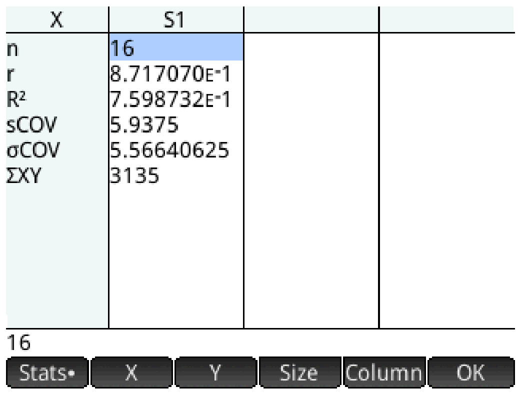
Pressing Fit on the Symb screen will turn the line on and off.



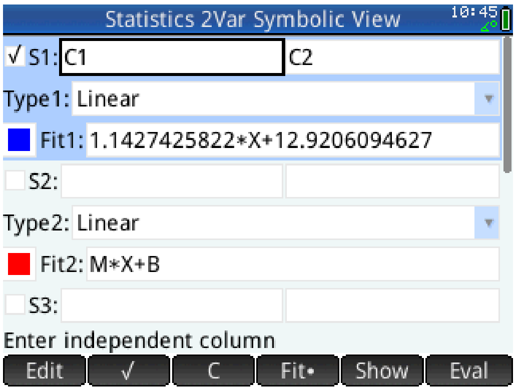
Press Num and Stats to see the summary statistics with the correlation coefficient and covarience.

Notably, the correlation coefficient $r = 0.87$ (t 2 d.p.) shows a reasonably strong relationship.

Press OK to leave the stats screen.



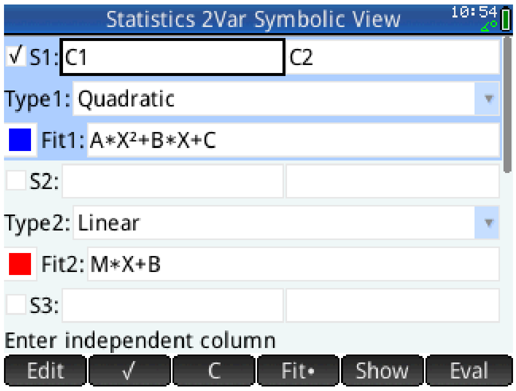
Press Symb again and notice that the details for the regression line are now shown.



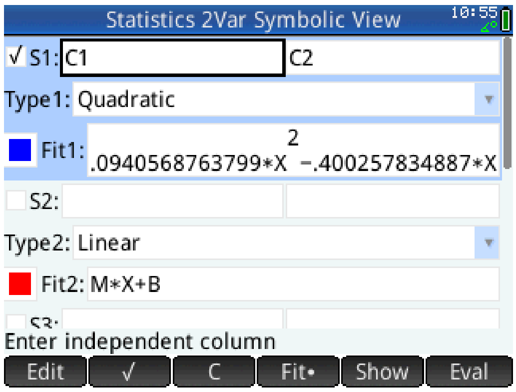
The line is $f(x) = 1.14x + 12.9$ (to 3 s.f.) and our x values (i.e. the first list) showed shoe size. This suggests a model of the relationship between show size and handspan as:

$H = 1.14S + 12.9$ where H =handpsan and S = shoesize.

If we want to see if we would get a better fit with a different model, then we can change the model with the Type options. Navigate to the Type1 entry and choose Quadratic.

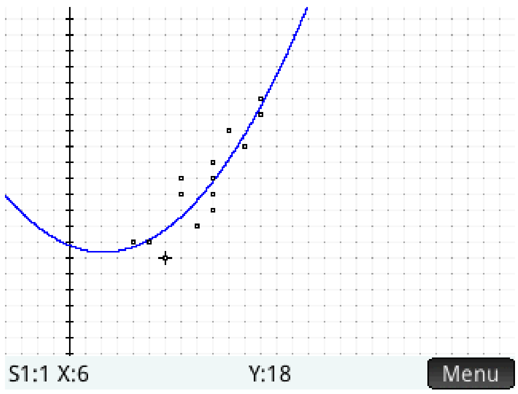


Press Num and Stats to calculate the statistics (many of which are undefined in this new model) then press OK and press Symb again to see the coefficients for the new model.



This gives us a new model of $H = 0.94S^2 - 0.4S + 18.8$ where H =handpsan and S = shoesize.

Press Plot to see the new model.



Confidence Intervals

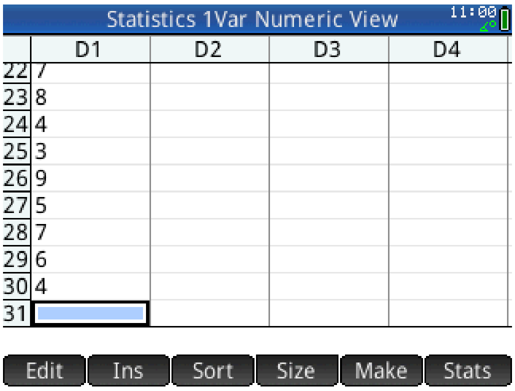
We have some data about the shoe sizes of thirty female maths students. We want to use this data to answer the question, “What is the mean shoe size for UK female maths students?”

Here we are using a sample to ‘infer’ something about the whole population. This is called inferential statistics. The issue will be how confident we can be in the outcomes. We can say that with a given level of probability, the mean will lie within a certain interval. At a 95% level of confidence, we can calculate the range within which the mean will lie, with a probability of 0.95.

To do this, we need to know the mean and standard deviation and the number of data values of the sample.

First, we enter the sample data into the 1 variable statistics applet. Press Apps, choose Statistics 1Var then Reset/OK/Start then enter the data into list D1

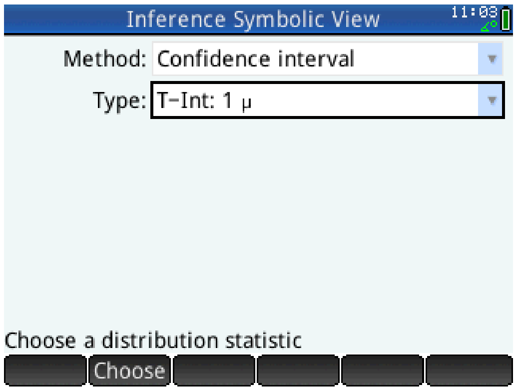
The data is: {2, 7, 6, 4, 5, 3, 4, 5, 6, 7, 4, 3, 5, 4, 4, 3, 5, 6, 6, 8, 6, 7, 8, 4, 3, 9, 5, 7, 6, 4}



Now start the inference applet:

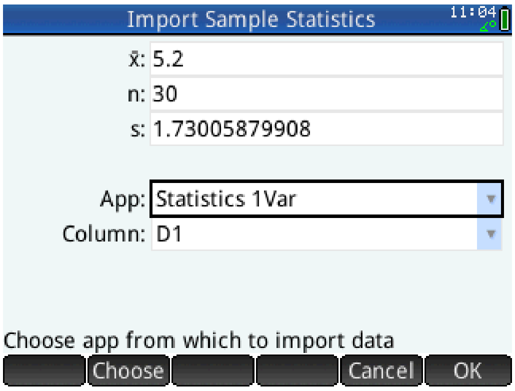
Press Apps, choose Inference then Reset/OK/Start

Chhose to change the Method to Confidence Interval and the Type to T-Int 1μ

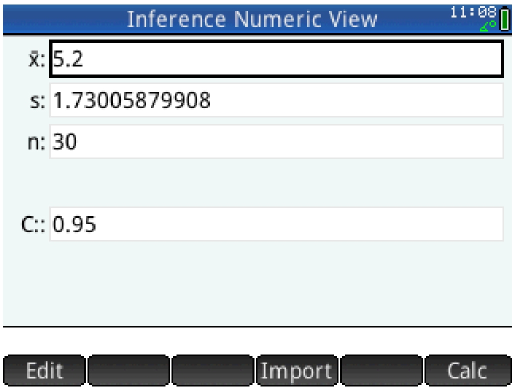


Press Num to get the statistics we entered earlier.

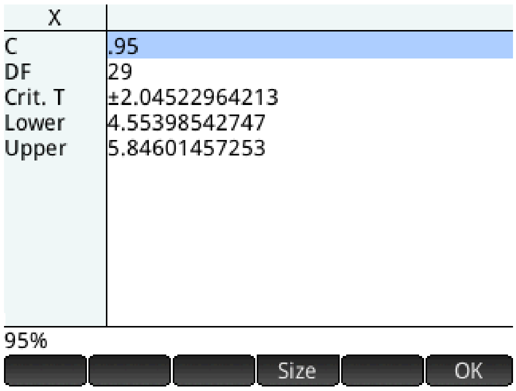
Press Import/ IMPRT. Check that we will import list D1 from the Statistics 1Var applet (which it will be expecting) and click OK



You can now see the sample mean $\overline{x}$, the sample standard deviation s and the number of sample data points n. You need to ch ange the confidence interval C to 0.95

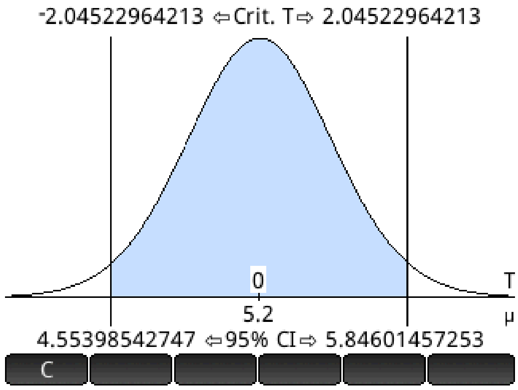


Press F6 for CALC to make the calculations.



This tells us that the probability is 0.95 that the shoe sizes of the whole population will fall between 4.55 and 5.85 (to 2 d.p.)

Press Plot to see this graphically.



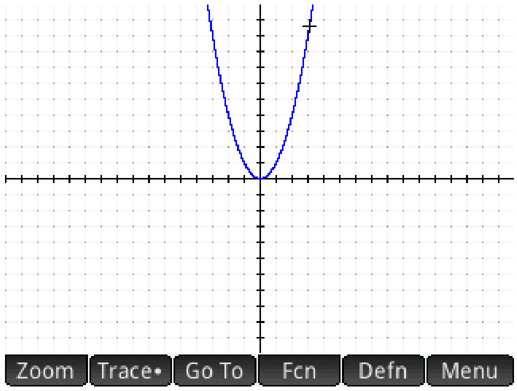
As usual, experiment! See what impact changing the confidence interval has. Look at the other tests available. Research them. See what difference they make.

8. Calculus

It is important to remember that the HP39gII is a numeric calculator. The HP Prime has both numeric and symbolic modes. This chapter lets you explore how calculus works using the numeric mode.

Differentiation

Let's explore the differential of $f(x) = x^2$. The differential is the function which gives values of the slope of the original function at all values of x.



Open and reset the Function applet:

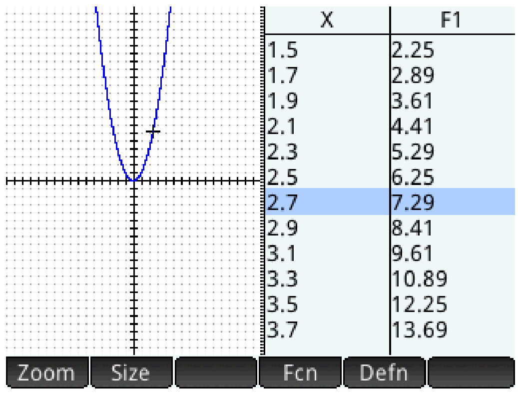
Apps/Function/Reset/OK/Start

Enter the function $f(x) = x^2$

Press plot to see the graph.

Make sure that Trace is on (the little blob is showing next to Trace) and move the cursor to the furthest right hand/positive end of the visible graph. Now press Fcn and choose Slope.

Before you move the cursor, guess what the slope values will be as you move the cursor to the left hand/negative end of the visible graph. When you have formed a good idea, move the cursor and see.

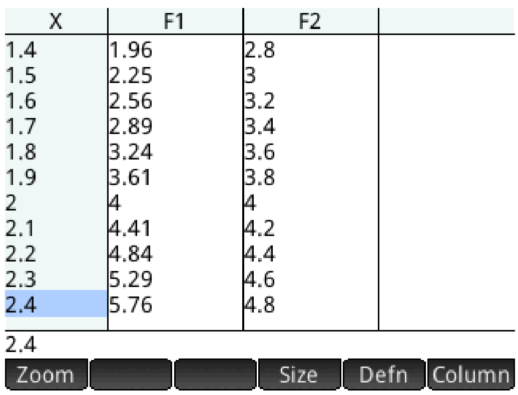


X	F1
1.5	2.25
1.7	2.89
1.9	3.61
2.1	4.41
2.3	5.29
2.5	6.25
2.7	7.29
2.9	8.41
3.1	9.61
3.3	10.89
3.5	12.25
3.7	13.69

Look to see the relationship between the value of the slope and the value of x. (It is easiest to see at key points like the one shown where the slope is 4 and you can see the value of x is 2).

You can see the relationship more clearly by splitting the screen. Press View/Views. Choose the second option (Split Screen: Plot table).

Press Fcn and choose slope, then choose another value of x in the table and find the slope.



X	F1	F2
1.4	1.96	2.8
1.5	2.25	3
1.6	2.56	3.2
1.7	2.89	3.4
1.8	3.24	3.6
1.9	3.61	3.8
2	4	4
2.1	4.41	4.2
2.2	4.84	4.4
2.3	5.29	4.6
2.4	5.76	4.8

Jot down the pairs of values if x and the corresponding slopes that you have found and decide what function shows the relationship between x and the slope. Go back to the Symb page and enter this function for F2(X). Press Num and check that the values for F2(X) are the same as the ones you jotted down.

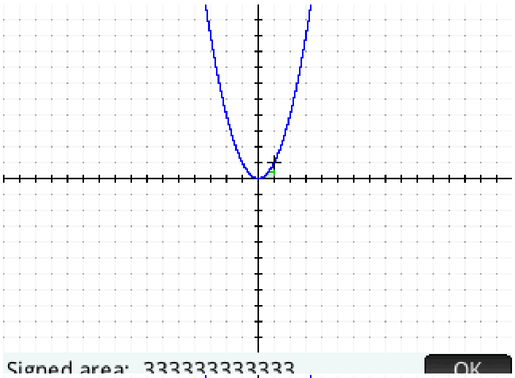
Repeat this process with a range of functions. The sine function is very interesting. Make slight changes to the functions and see what difference it makes.

(e.g. change $f(x) = x^2$ to $f(x) = x^2 + 1$)

Get a good feel for the differential functions generated by different types of functions.

Integration

You can apply the same process to get a feel for the integral function. This time we are looking at the area under the graph.

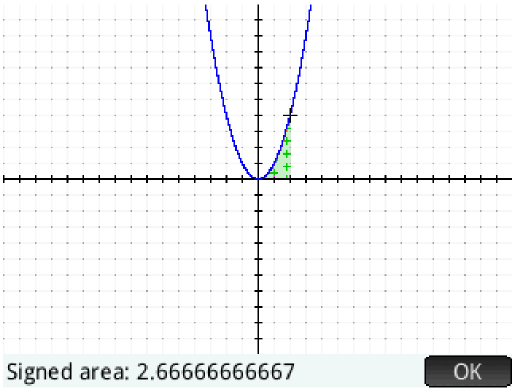


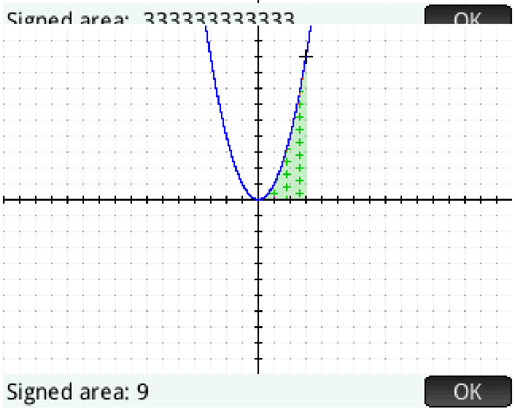
Launch and reset the function applet.

Enter the function $f(x) = x^2$ and Plot it.

Press Menu then Fcn and select Signed area. The cursor will already be at the origin, if not, move it there. Click OK. Now move the cursor to $x = 1$ and click OK.

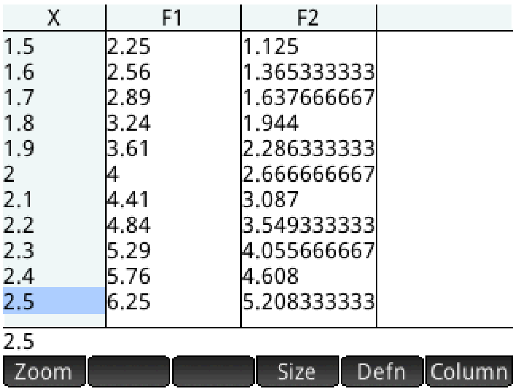
Now repeat the process starting from $x = 0$ to $x = 2$ … then from $x = 0$ to $x = 3$





Writing the values as fractions makes things clearer:

x	0	1	2	3
Area from 0 to x	0	$\frac{1}{3}$	$\frac{8}{3}$	$\frac{27}{3}$



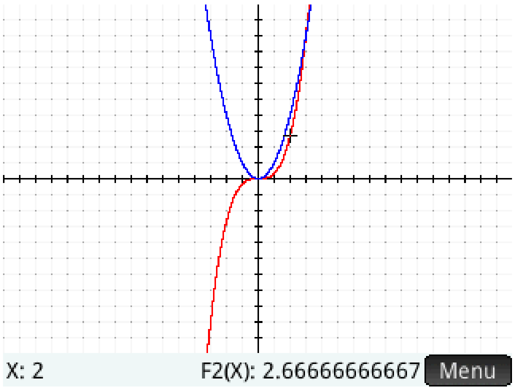
X	F1	F2
1.5	2.25	1.125
1.6	2.56	1.365333333
1.7	2.89	1.637666667
1.8	3.24	1.944
1.9	3.61	2.286333333
2	4	2.666666667
2.1	4.41	3.087
2.2	4.84	3.549333333
2.3	5.29	4.055666667
2.4	5.76	4.608
2.5	6.25	5.208333333

2.5

Zoom Size Defn Column

Decide what you think the area function (the integral) is, Press Symb and enter it as F2(X). Press Num and check that this generates the same values.

Press Plot and TRACE over F2(X) to check.



Again, do this with different types of functions. Make small changes to existing functions. The aim is to get a feeling for how the area function varies. To be able to work out exactly what the function is from this small amount of data would be very difficult for most functions. But the key point is to experiment and get that all important feeling for how calculus works.

Notes

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www.hpgraphingcalc.org

The Maths Activities Pack

Using HP Prime and HP39gII Graphing Calculators

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Introduction

Graphing calculators have been around since the late 1980s. They are easily carried, turn on instantly, are extremely reliable, yet have software as powerful as anything available on a full computer for doing mathematics. In the classroom, ICT use is often difficult to arrange, booking the room, finding enough working machines and keeping the students off the internet are ever present problems. With a class set of graphing calculators in a small, easily carried box and a software emulator on the classroom computer, the teacher can have proper technology rich lessons with minimal set up time and maximal reliability. It just works!

Hewlett Packard have been in the forefront of calculator technology since its earliest days. They are renowned for the very high physical quality of the machines together with the most thoughtfully developed software. Sometimes, teachers are worried about the complexity of graphing calculators, but HP's designs work the way maths teachers think. Ease of teacher use is built in to the design from the start.

The activities in this booklet are designed for use with the HP Prime and HP39gII with additional activities also incorporating the StreamSmart data logging device. Both of the calculators have extremely comprehensive scientific functions, including matrices and complex numbers. However, their power is seen in the range of function types for which multiple representations can be set up. There is considerable research evidence showing the improvements in students' understanding when they have access to symbolic, graphical and table representations. The HP Prime and HP39gII make this a central feature and can quickly show all three, for functions plots, polar graphs, parametric graphs and many more. The same structure even applies to one variable and two variable statistics. Sets of side by side box-and-whisker plots can be drawn as soon as the data is entered, together with the summary statistics. Students don't need to laboriously draw the charts themselves, instead they can get straight to the statistical analysis.

HP Prime has a computer algebra system (CAS). This means that it can operate on algebraic statements, working symbolically. It will factorise symbolic expressions, it will integrate and differentiate symbolically and very much more. The CAS can be disabled securely so that the machine can still be used in public examinations.

The HP39gII has a high resolution screen able to show grey scales for beautiful, smooth graphs, coupled with a fast processor and more than enough memory. Also, a sophisticated battery management system means that standard AAA batteries can last for months, meaning you can always be sure of a complete set of working machines. There are very many ease-of-use improvements which the user will see in operation. Options will come up when they are needed. There are six SoftKeys F1 to F6, which make navigation much clearer. Also, there is a comprehensive notes system for developing classroom activities and a detailed, context sensitive help system.

HP Prime is a major advance. Taking all of the features of the HP39gII, plus the CAS system, there is now a multi-touch full colour screen for a fast and intuitive interface. Then there is extra software, a dynamic geometry system, a spreadsheet and the amazing advanced graphing app, which can graph ***any*** statement in x and y.

The StreamSmart device together with data logging probes plugs directly into the calculators and allows very fast, real time logging of a variety of data types. For example, students can log their displacement from the device, creating real-time distance-time graphs. There is a thermometer, a microphone, a pressure probe and a light intensity probe available, allowing for a range of experiments for STEM lessons.

The activities in this booklet are not set out as lesson plans. We assume that teachers will want to adapt them for different parts of lessons and different timings. Instead we have given details of the mathematics and instructions for conducting the activity, together with commentary on classroom use. All of these have been used by the author in perfectly ordinary secondary school classrooms.

The emphasis is on exploratory and investigative working. The power of the calculator lies in its ability to give instant feedback to ideas and to generate data very quickly. Students can generate data, make and test hypotheses about how mathematical operations and ideas work. They then test and develop their hypotheses to reach a conclusion. At this stage, the technology must be set aside and the student can practice their ideas away from the technology.

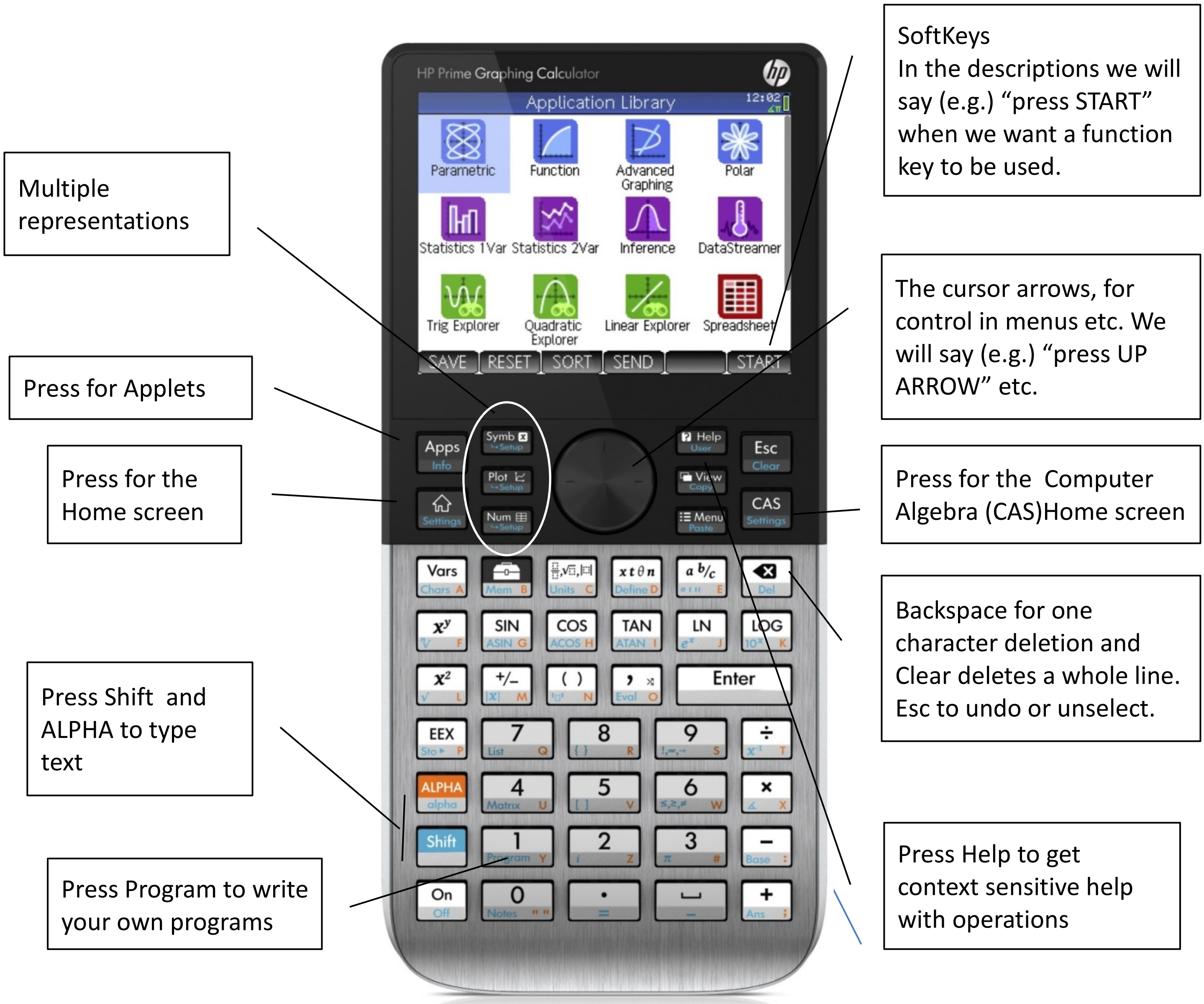
We would recommend a standard class set of one between two students, to maximise collaboration and discussion. However, there is strong evidence to suggest that students owning their own machine, personalising it and developing their understandings over time, brings the maximum benefit in the long term.

We would welcome your feedback and suggestions, whenever you try any of these activities with your students, to help us develop future editions of this booklet. Hewlett Packard is proud of the support that it offers and you should be confident to ask for any support you may need. Manuals, guides, new downloads and details of training and support is available at www.hpgraphingcalc.org.

Chris Olley
Updated October 2013

Getting Started with HP Prime

First, remember, HP Prime has a multi-touch touchscreen. Look for options and opportunities on screen to press to get started. To start calculating, press **Home**. Then, notice there is no = key. Use the **Enter** key to evaluate.
If you get stuck, press the **Home** key to get back to the home screen. Press Esc, to deselect or move back in the menus. Press the **Apps** key, to return to the opening Apps menu. Press this to find all of the built in applications. This is the central place for advanced functions and graphing.
Find the **Symb**, **Plot** and **Num** keys. These allow you to instantly switch between the three representations: symbolic, graph and table.
The bottom of the screen shows SoftKeys. Press these on screen. In the text we will refer to these by name, for example, we will say, “press START”. These are the context options (like right clicking in computer software), so look here for options.



Getting Started with HP39gII

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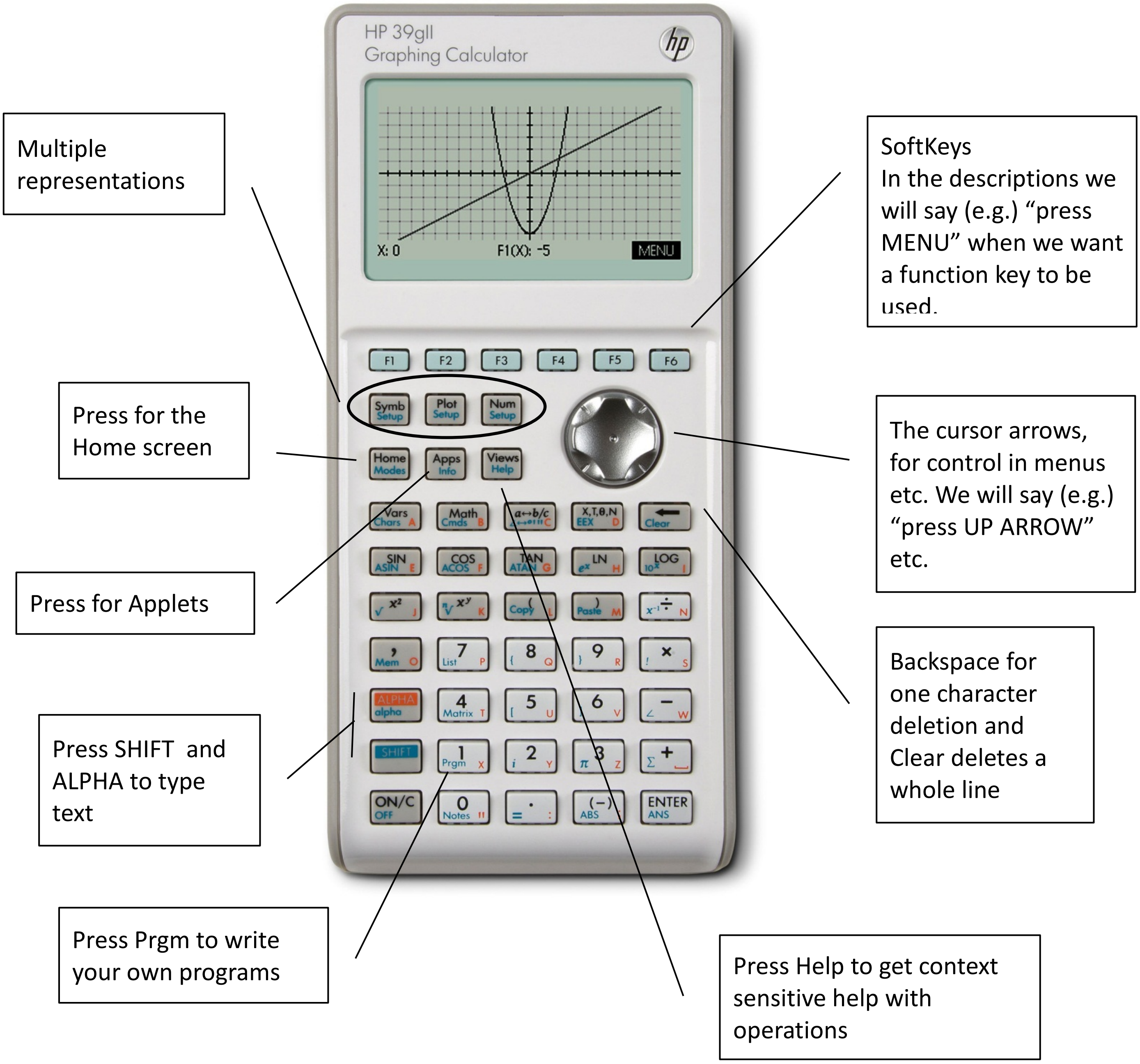
Now find the **Home** key. If you ever get stuck, press the home key to get back to the home screen.
Now find the **Apps** key. Press this to find all of the built in applications. This is the central place for advanced functions and graphing.
Find the **Symb**, **Plot** and **Num** keys. These allow you to instantly switch between the three representations: symbolic, graph and table.
The top row of 6 HP39gII keys are SoftKeys. Look at the bottom of the display. Small black rectangles above each SoftKey show what the SoftKey will do. In the photo, pressing the right most SoftKey will show the graph menu. On the screen shown to access the menu press F6 … we would say “press MENU”.



1: Number Explorations

It is important to get students started with an activity which will make them comfortable with the machine. For any new group I have always started with a lesson asking students to explore the different functions that they can find. They can be guided to focus on the two rows of keys starting with SIN and X^2 and those below. I have found that if the teacher continues to prompt students simply to keep trying out new numbers and new functions, then interesting discoveries are made. Students can present their findings at the end of the lesson. In my experience, in very ordinary schools, key stage 3 students will say things like “that SIN button, doesn’t matter which number you try, the answer is always less than 1”. Some students will explore lots of features and the teacher may need to keep them focussed on one interesting idea or may be prepared to let them try out anything!

Activities for Exploring Properties of Number

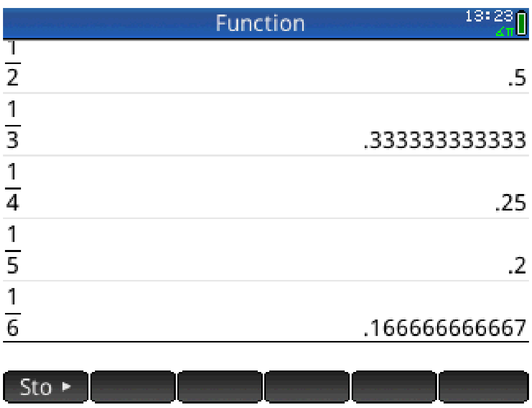
A very powerful use of this technology is to quickly generate lots of data from which students can draw inference. For me, it is much better if students can work out how mathematical processes work and then report their thinking to the teacher and their peers, rather than the teacher tell them their way of doing things. The learning is then much more secure and expressed in terms the student understands because they did it themselves. I use a set of simple activities (which nonetheless can generate some very thoughtful mathematics), to help students get started with this way of working. These activities could all be done just as well on a scientific calculator, but the aim here is to develop a way of working which can then take advantage of the advanced possibilities of the graphing calculator.

Exploring decimal representations of fractions

Press: Shift Settings/Modes. Make sure the *Number Format* is set to standard. Then press Home.

Now enter fractions to explore e.g.

1 ÷ 2 Enter
1 ÷ 3 Enter
1 ÷ 4 Enter
1 ÷ 5 Enter
1 ÷ 6 Enter
etc.



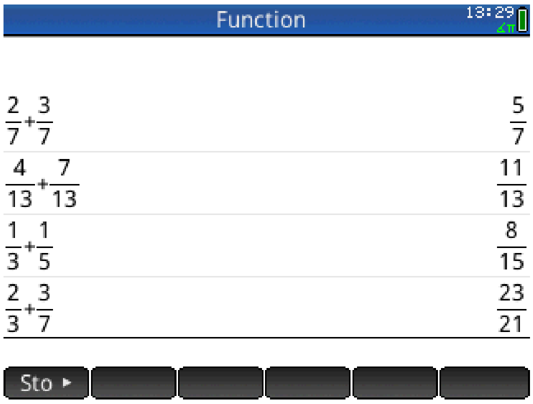
Students can be asked to conjecture in each case. The order given is intriguing because it suggests a pattern which does not emerge. Students can look for relationships e.g. between 0.5 and 0.25 and 0.33333… and 0.1666… Important details such as the suppression of trailing zeros and final rounding (1 ÷ 6 = 0.1666….67) become issues for discussion. Then students can explore with the calculator to find families of fractions and relationships between their decimal representations.

Exploring addition of fractions

Use the $a \leftrightarrow b/c$ key to show the result as a fraction.
On HP Prime after entering the first fraction, press *Right* to enter the operation.

Now enter added fractions to test

- 2 ÷ 7 + 3 ÷ 7
- 4 ÷ 13 + 7 ÷ 13
- 1 ÷ 3 + 1 ÷ 5
- 2 ÷ 3 + 1 ÷ 5
- 2 ÷ 5 + 3 ÷ 7



Students should write the correct form of the output into their notebooks. $\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$

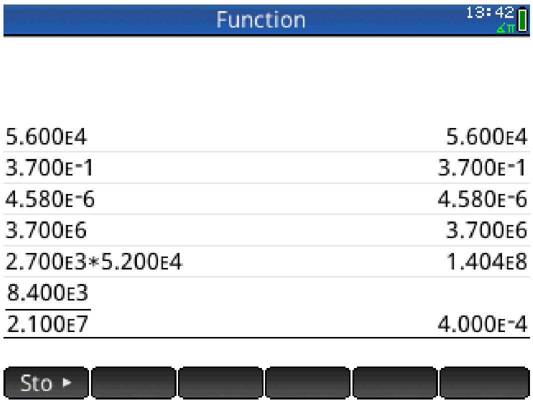
The examples given are chosen to support students to explore each type thoroughly. They should try different examples of each type, make and then test conjectures about how the fractions are being added. They can then move on to the next type to see if their conjecture still holds or if they need to develop it further. In the end they should have a secure mechanism for adding fractions which they can explain and exemplify.
Note that the calculator will always simplify fractions, so it may be necessary to guide students to examples which allow them to investigate the effect of this.

Exploring Standard Form

Make sure the number mode is set to scientific and fixed to (say) 3 d.p.

Shift Settings/Modes Choose Scientific $\rightarrow$ 3. Then press Home

e.g. 56000 Enter
0.37 Enter
0.00000458 Enter



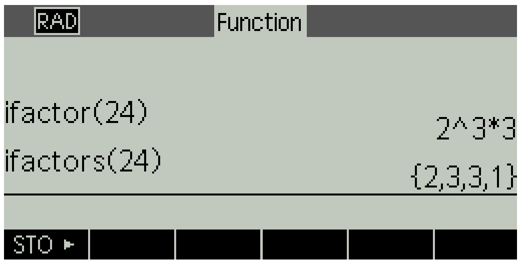
Write the results in standard form:
The calculator reports 56000 as 5.600E4
Students should write $56000 = 5.6\times10^4$

then: 370 shift EEX 4 Enter

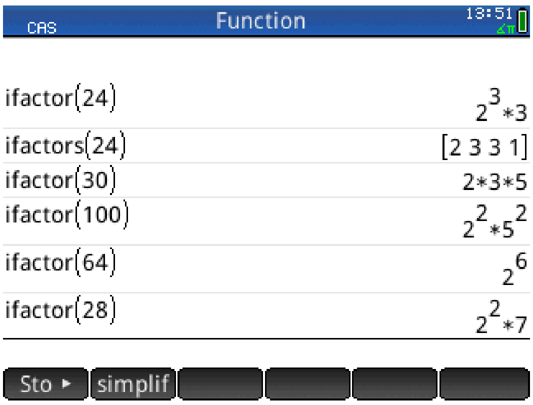
Again, students should generate sufficient data to be able to describe their process and give examples in each case (convert into standard form, convert from standard form, numbers greater than 1 and numbers between 1 and −1 and finally arithmetic on numbers in standard form). As always it is important for the students to write their data, correctly formatted into notebooks. This allows a clear focus on accurate written mathematics.

Exploring Factors

By generating lots of data from prime factorisations students will develop an idea of prime numbers as those with exactly two distinct factors and how only these numbers can be used to make up all other numbers. They can develop this work by investigating the effect of the Factor command on algebraic expressions.
The objective is to develop an understanding of prime numbers, methods for prime decomposition and the process of factorisation in general.



We use the ifactor (for integer factorisation) command in the Math menu.
On HP39gII:
Press Math and navigate to Integer then ifactor type a number, say 24 and close the bracket. Press Enter.
You can also see a list of the factors using the ifactors command.



On HP Prime, the ifactor command is a CAS command. You can use this in the Home screen, but it is better in the CAS screen. So, press the CAS button. The press the Suitcase button and choose Integer/Factors (in the CAS tab). The list of factors is available with Factor List in the same menu.

This yields the result $24 = 2^3\times3$

2: Algebra Explorations

Exploring algebraic expressions
[Note **Sto** is the left SoftKey on the home screen]

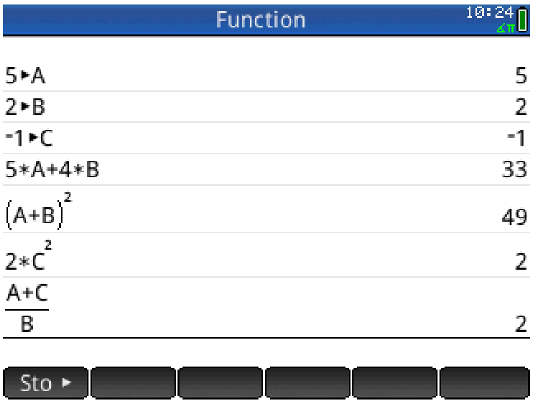
The idea here is to get students to experiment with the values of expressions. They choose values for (say) 3 different letter variables.

Set some values for A, B, and C e.g.

5 STO ALPHA A Enter

2 STO ALPHA B Enter

+/- or (-) 1 STO ALPHA C Enter



Students can then type different expressions to test. This works well as a paired activity. Students are allowed a maximum of (say) 5 characters to type an expression. Their neighbour has to say what the value of the expression is, before the first student presses the Enter key to evaluate.

Now test different expressions (for example)

5 ALPHA A + 4 ALPHA B Enter

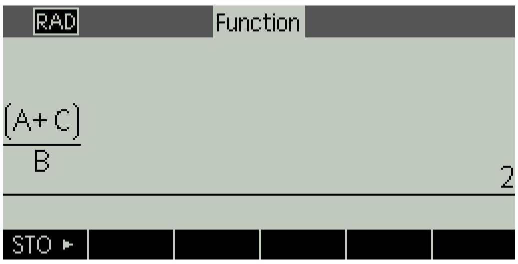
(ALPHA A + ALPHA B) X^2 Enter

2 ALPHA C X^2 Enter

ALPHA B ^ ALPHA C Enter

(ALPHA A + ALPHA C) ÷ ALPHA B Enter

(HP39gII) To show the algebra correctly SHIFT/Modes/Page down/Textbook display



Students should copy examples into their notebooks.

Place Value and the Algebraic Symbol System

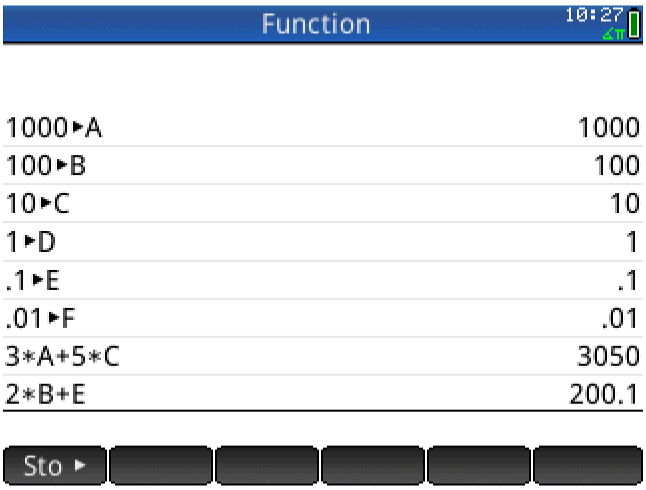
This activity gets students to explore place value by setting up expressions. First, the letter variables A to F are defined as a suitable set of powers of 10.

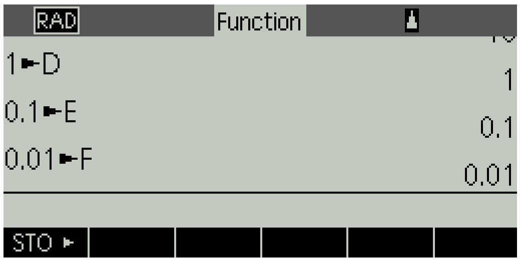
Press 1000 Sto ALPHA A Enter
Press 100 Sto ALPHA B Enter
Press 10 Sto ALPHA C Enter
Press 1 Sto ALPHA D Enter
Press .1 Sto ALPHA E Enter
Press .01 Sto ALPHA F Enter

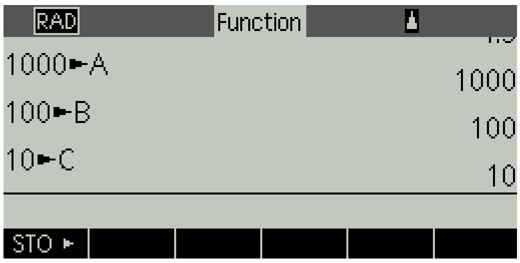
A is 1000
B is 100
C is 10
D is 1
E is 0.1
F is 0.01

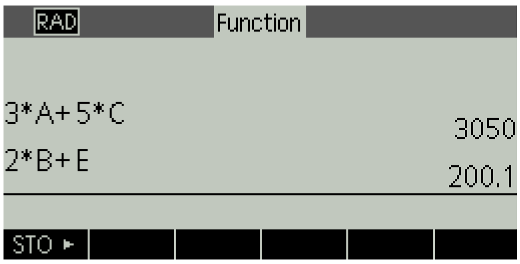
We suggest that the teacher does this on the board before students have access to the calculators themselves. They can then have a worksheet to fill in like the one on the next page. Students can be allocated a calculator when they have filled in the 'My Idea' column for every question. They then follow the instructions to set up the calculator and check, filling the calculator response into the end column.

After this students working in pairs can set and test challenges for each other as before.









Place Value and the Algebraic Symbol System Worksheet

Press 1000 Sto ALPHA A Enter
Press 100 Sto ALPHA B Enter
Press 10 Sto ALPHA C Enter
Press 1 Sto ALPHA D Enter
Press .1 Sto ALPHA E Enter
Press .01 Sto ALPHA F Enter

A is 1000
B is 100
C is 10
D is 1
E is 0.1
F is 0.01

1.		My Idea	Calculator check
	3A		
	5C		
	2B		
	8E		
	A + B		
	2B + C		
	5C + D		

2.		My Idea	Calculator check
	D + E		
	3C+2E		
	A+F		
	B+2C+3D		
	5B+D+3E		
	6A+3D+F		
	3C+D+E+7F		
	6A+E+F		

3.		My Idea	Calculator check
	700		
	0.8		
	15		
	5020		
	307.6		
	5004		
	12.83		
	5070.09		

3: Exploring Calculus with CAS (HP Prime only)

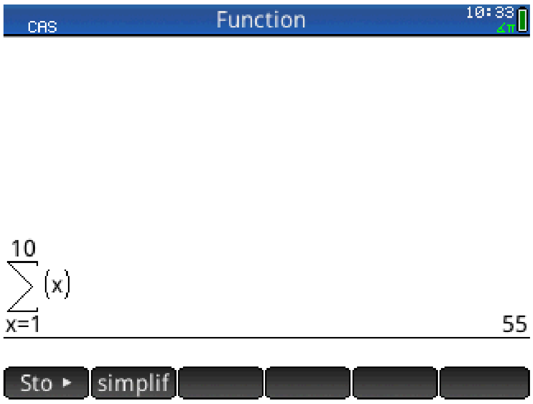
Having a machine which can find do the algebra for you is very powerful. Clearly, if an exercise question (or exam) requires a student to just do the algebra, then the machine cannot be used. So, we have two principal opportunities:

- Use the calculator to provide plentiful data to explore how the algebra is being done.
- Support more complex tasks by giving confidence in the algebra, to free up thinking time on the mathematics.

We will be using the CAS functions in this section. So, on your HP Prime press the CAS key to open the CAS home screen.

Start by exploring the sums of series. Use the template key to insert a summation template.

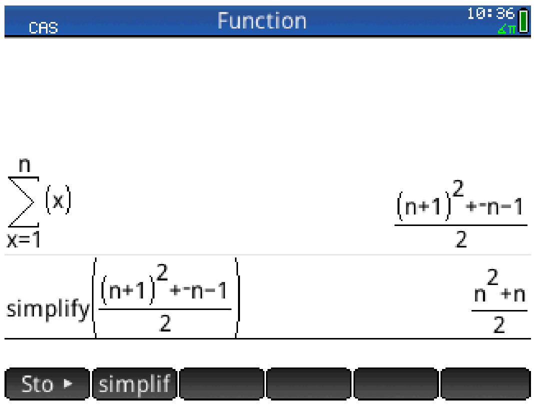
Use the left and right cursor keys to navigate to each entry point in turn, to input a suitable sum. Use ALPHA to choose your variable. (*Notice that CAS variables are always lower case, so HP Prime defaults to lower case when ALPHA is used in the CAS screen*).



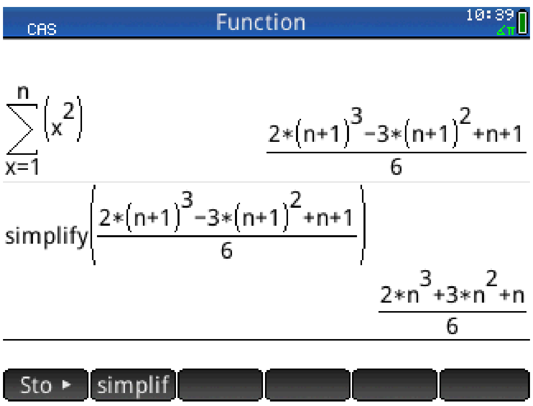
Press enter to generate the solution (55)

Now repeat to find the general expression for the sum to n.

Unsimplified solutions are always displayed first, so keep pressing **simplif** to simplify further.

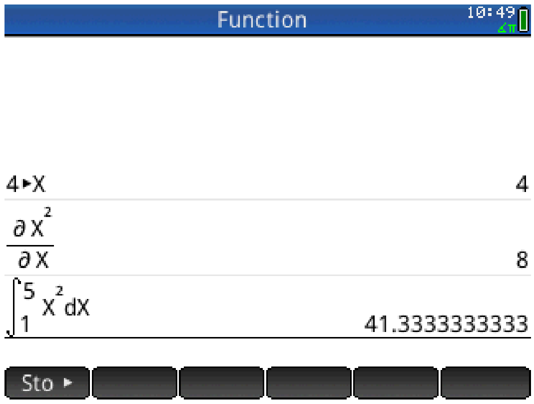


Press up to highlight a past entry, then Copy. Now make some change to the expression and see what difference it makes. Form conjectures and test them.



You will also have seen calculus templates available from the template key.

On the numeric home screen we can work numerically. e.g. choose the d/dx template, Enter X as the variable and X^2 as the function. It will evaluate the differential at the current value of X. Use Sto to change the value of X. In this screen I have used Sto to set X=4 then entered the differential.

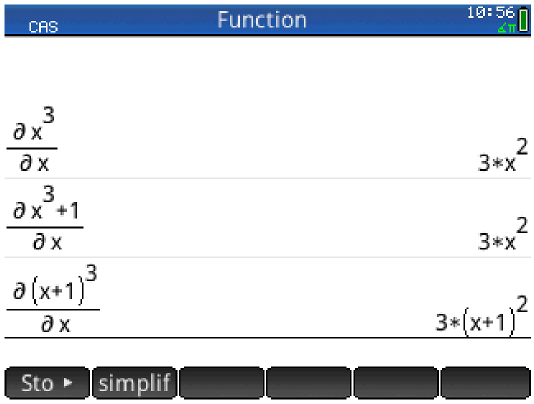


Use the integration template for numeric integrals, for example Enter: $\int_1^5 x^2 dx$ and it will be evaluated using the current value of X. (Here it is still X=4 entered earlier.

Press CAS to return to the CAS home screen and explore the calculus commands again.

Using the d/dx template, type in suitable elements using the cursor. Then press Enter.

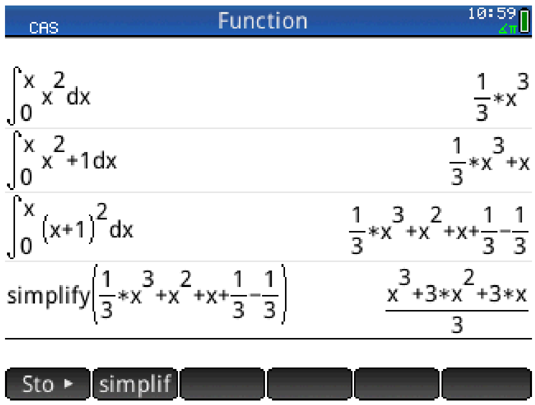
(Press simplif to simplify if needs be).



Students can now explore how the sums, differentials and integrals change with changes of function and with changes of values and limits. They can categorise calculus methods.

How open or how structured their exploration, is down to the teacher's judgement, but the calculator provides any amount of data needed.

Here is an example of integration. Experiment with different settings for the limits, to handle the constant of integration.



4: Exploring Graphs

The graphing calculator naturally offers very quick and easy to use graphing software. Most graphing is controlled from the top row of labelled keys. These give quick access to the *multiple representations* of functions: algebra, graph, table. These are the SYMB, PLOT and NUM keys.

HP Prime: 

HP39gII: 

Notice that Setup options are available for each view by pressing shift plus the appropriate key.

You will want to put a variable in when you define a function. You can use the ALPHA key to choose any letter. However, there is a $x\ t\ \theta\ n$ (HP Prime) or X,T,θ,N (HP39gII) key, which chooses the correct variable for any given graph type. Also, an on-screen SoftKey is available for the standard variable.

By default, the calculator graphs function plots $y = f(x)$. (You will see 'Function' showing in the top line of the screen, to show that the Function App is running). However, there are many other types of function which can be plotted. Press the Apps key to see what is available:

- Function
- Parametric
- Polar
- Sequence

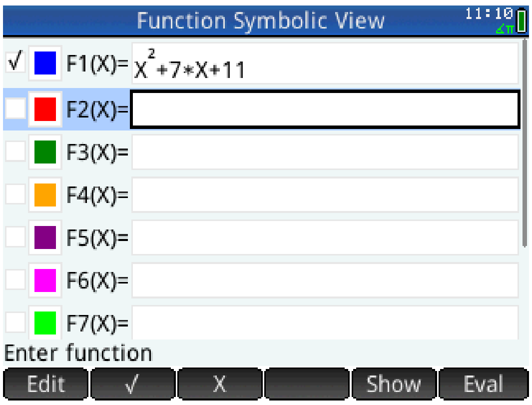
You can store a number of functions of each type and keep them in the calculator's memory, switching between the different types using the Apps key. We suggest that whenever you start a new activity, you RESET the applet before starting. This way all of the settings will be returned to normal and any graphs from an earlier user will be deleted.

Try this out to get started:

Press Apps navigate to Function and press Reset then OK then Start

[This achieves the same effect as pressing the Symb button on a new calculator]

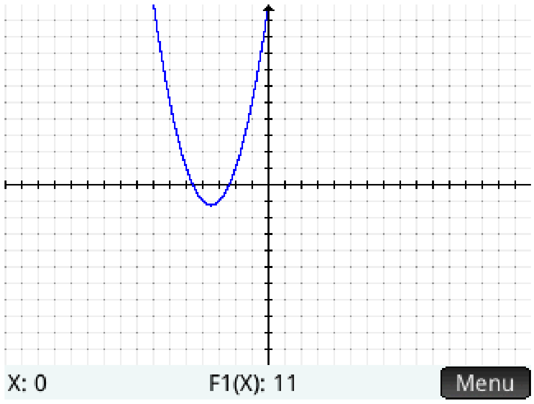
Type a function (say) $X^2+7X+11$ and OK



Press the Plot key.

Notice the SoftKey Menu. This gives access to all of the graphing facilities.

- You can Zoom in on parts of the graph.
- You can Trace to see values of points on the graph.
- The Fcn key allows roots, turning points, slope, etc. to be found.

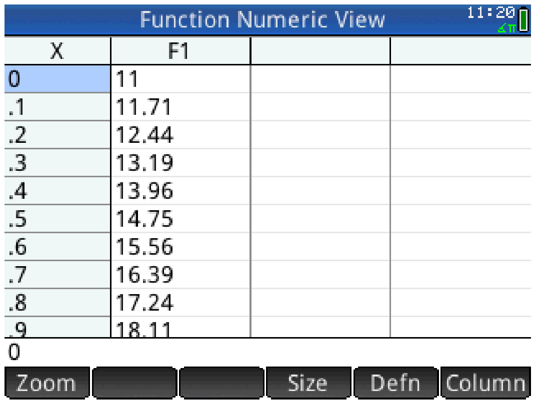


A fast zoom is available by pressing the + and – keys. This zooms in and out using the current position of the cursor as the centre. Use this with care!

Press the Num key.

You can navigate up and down the table to look for (say) turning points or points of intersection.

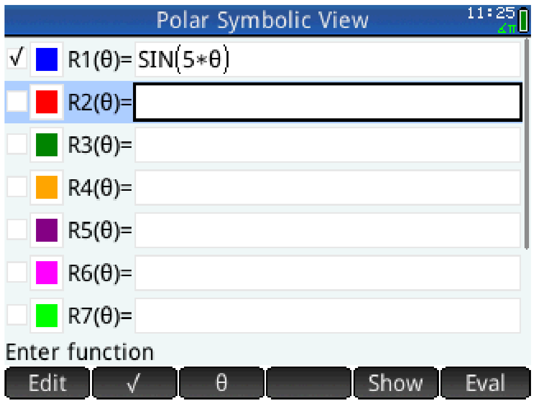
You can even zoom in the table, to find (say) roots to greater levels of accuracy.



Polar Graphs

Now press Apps, navigate to Polar and press Reset then OK then Start.

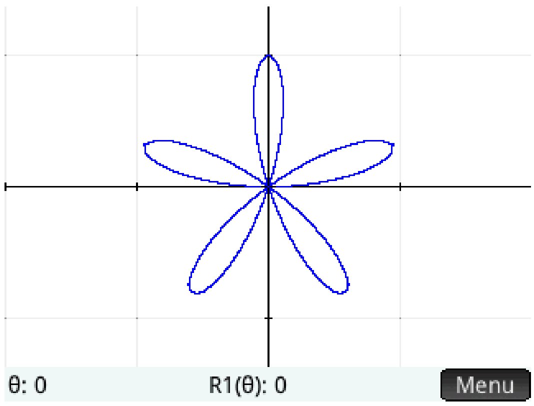
Notice the function definitions are now in polar form. Type sin(5θ). The $x\ t\ \theta\ n$ key now inserts a θ instead of an X. Press OK.



Press PLOT to see a tiny graph.

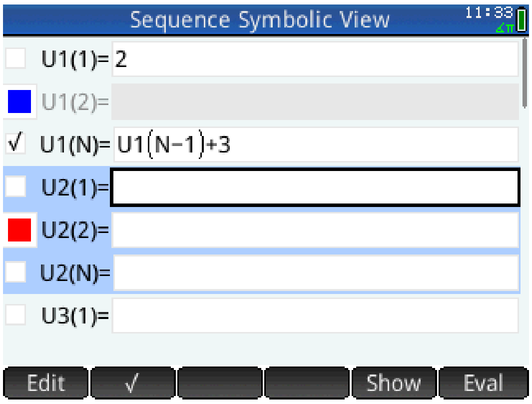
Press the + or – keys to get a good view.

Press Apps, navigate to Function and press Start (**Do not** press Reset!) You will see that your previous graph is still available as you left it.



Sequence Graphs

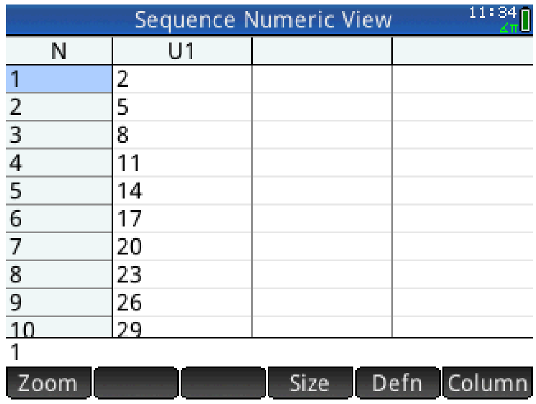
Press Apps, navigate to Sequence and press Reset then OK then Start.



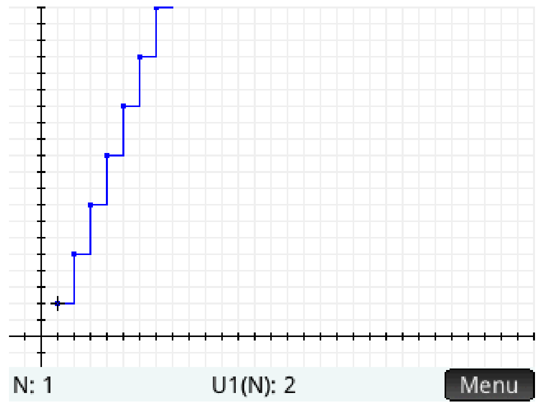
You can enter the first number in the sequence U1(N) and optionally, a second number in the sequence U2(N) (useful notably to enter a Fibonnaci sequence). Then enter a rule for the sequence, either in term-to-term or position-to-term form. Notice that when you move to the U1(N) line the on-screen SoftKeys give the building blocks required.

Here, we have given a simple sequence starting at 2 and with each term being the previous term plus 3.

Pressing Num shows the sequence.

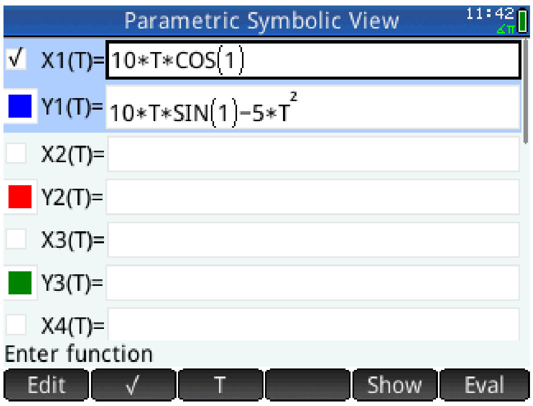


Pressing Plot gives a rather different view, which provides new materials for comparisons and a clear route to discussing the move from discrete to continuous functions.



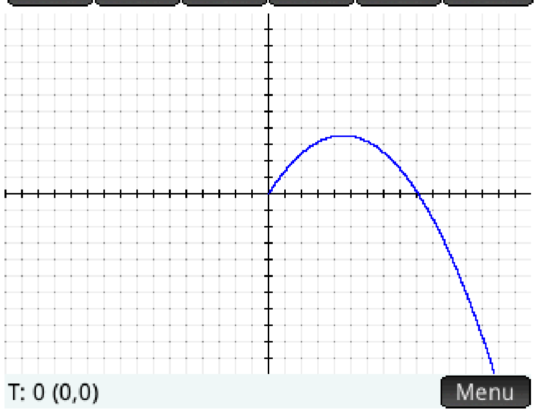
Parametric Graphs

Here is a classic example of a parametric graph:



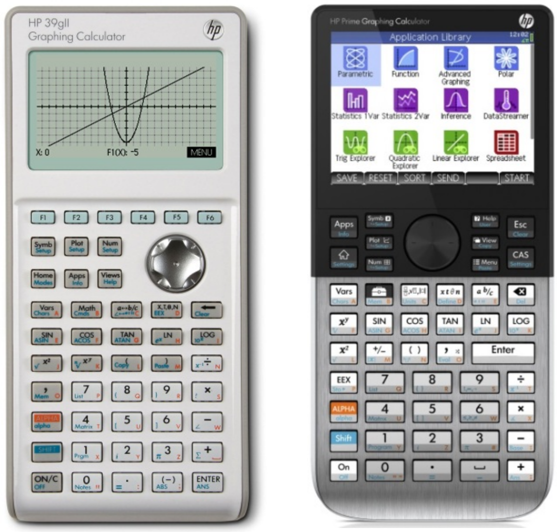
You should experiment with all of the different graph and function types. Try out different Zoom's. See what Setup options are available in each applet and in each representation. Your students now have an extremely flexible tool to explore functions of almost any type choosing the most suitable representation

to solve the problem at hand.



The next two pages contain a worksheet you can use which structures a student centred investigation into the effect of changing the coefficients on the graph of a quadratic function.

Investigating Quadratics Worksheet



A Quadratic Equation has the form

$ax^2 + bx + c$

These are all quadratic equations

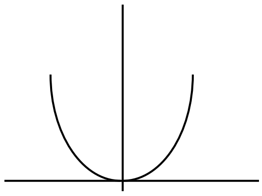
$y = x^2$

$y = x^2 + 3x$

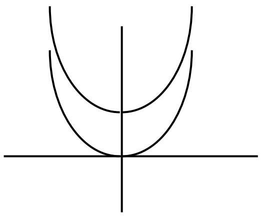
$y = x^2 - 4$

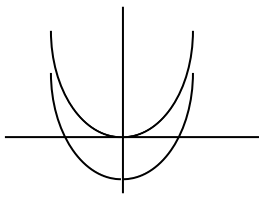
$y = x^2 + 2x + 1$

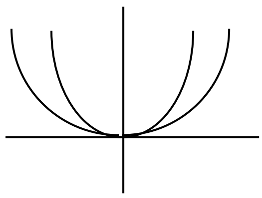
1. Graph $y = x^2$



2. Make graphs similar to these.
Write down the equations you have used.

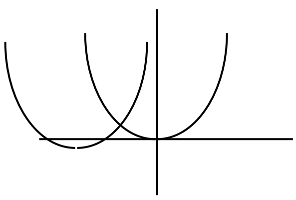


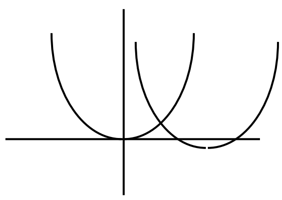


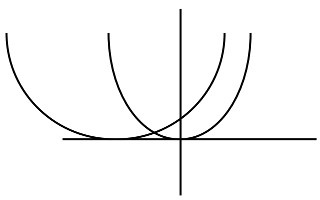


Make as many pairs as you can for each graph.

3. Make graphs similar to these.
Write down the equations you have used.







Make as many pairs as you can for each graph.

4. Write notes to explain how a graph of $y = x^2$ can be changed:
(a) to move up or down the y axis.
(b) to move left or right on the x axis.
(c) to get wider or thinner.

Hint: Try $y = (x - 1)^2$

5. Draw graphs of $y = x^2 + 2x$ and $y = x^2 - 4x$.

   Draw more graphs, changing only the coefficient of x.

   Explain the effect of changing the coefficient of x.

> *Note*: the number that we multiply the x by is the called the *coefficient* of x.

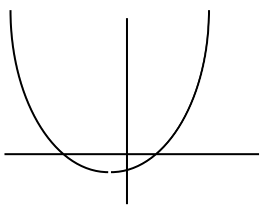
6. We can write a quadratic equation like this: $y = (x - 3)(x + 5)$

   Multiply out the brackets and write down the expanded form of the equation.

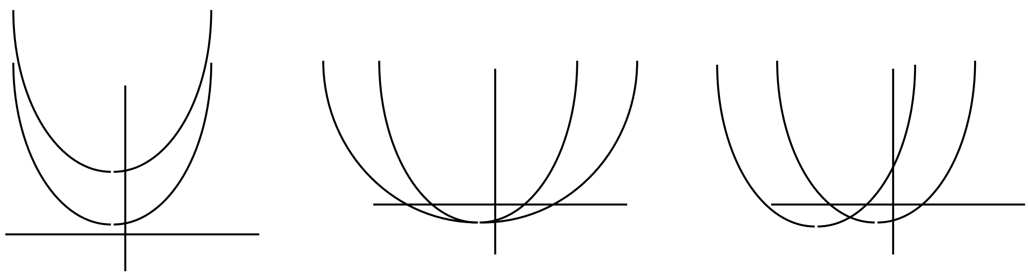
   Try out equations of this type. Change the numbers. Change the + and –.

   Explain what effect the numbers 3 and 5 have on the position of the graph.

7. Draw a graph of $y = (x - 3)(x + 5)$



   Make graphs similar to these. Write down the equations you have used.



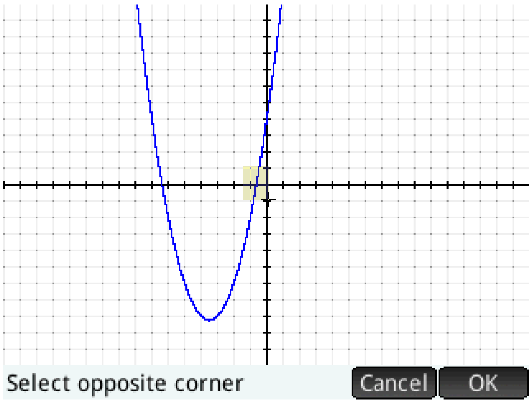
   Make as many pairs as you can for each graph.

8. Write notes to explain how a graph of $y = (x - 3)(x + 5)$ can be changed:
   (a) to move up or down the y axis.
   (b) to move left or right on the x axis.
   (c) to get wider or thinner.

Quadratic Roots

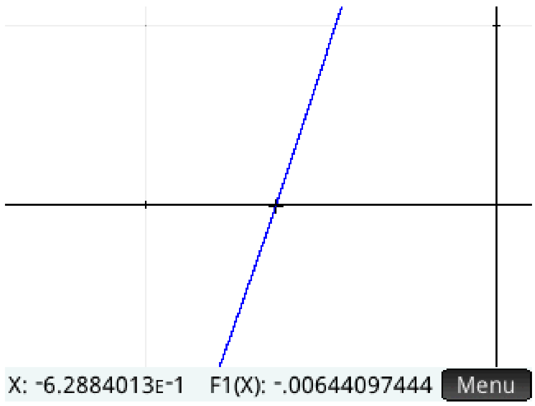
Try this activity yourself to consider the advantages of different methods for finding roots of a quadratic.

Method 1



Press Apps, navigate to Function and press Reset then OK then Start
Type X² + 7X + 4 then press OK (You can use X,T,θ,N for the X *then* X²)

Press Plot then Menu then Zoom.
Choose **Box…** then OK
Press on the screen (HP Prime) or move the cursor so that it a little to the left and a little above one of the roots. Press OK.
Now press again or move the cursor a little to the right and a little below the root. You will see a box formed around the root. You should aim to make a very small box with the root in the middle. If it all goes wrong you can press Zoom again and choose Un-zoom. Now press OK to complete the zoom.



Now you can press **Trace** and move the cursor as close as possible to the root, giving its value. Use Un-zoom to go back to the original graph and find the second root.

Method 2

Now find the solutions from a table of values for this function.

- Press Num and use the up/down cursor to explore the table of values.
- Find which two numbers one of the solutions of $x^2 + 4x + 7 = 0$ falls between.
- Press Shift Num and you can change the intervals in the table.
- Select Num Step and type 0.01 then OK then press the Num key.
- Repeat this to find both solutions correct to 2 decimal places.

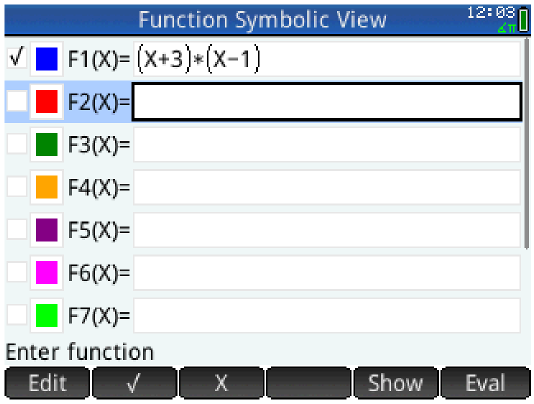
Method 3

Now get the calculator to find the roots.

- Press PLOT MENU FCN ROOTS
- This will show the root closest to the cursor
- Move the cursor with the arrow keys closer to the other root
- Press MENU FCN ROOTS
- This will show the other root

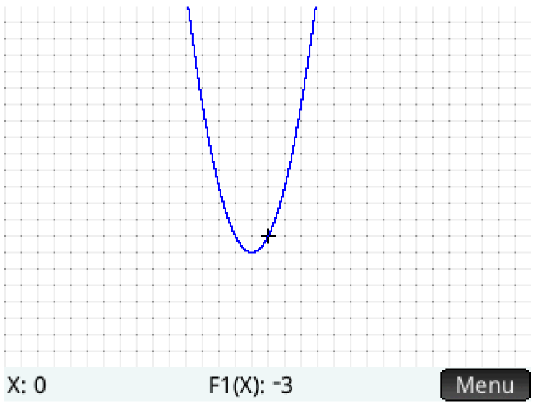
5: Guess the Function

This activity is ideal to get students to feel the variability in the function. It is really important to avoiding prompting but to leave students make guesses and get them wrong! As the amount of data increases, they will get better in their ability to see the outcomes. The teacher sets up a function unseen and students guess the output for different inputs. Students are in complete control of the inputs, but the teacher gradually helps them by increasing the views they can see. First, the values of *x* can be sorted to be in order. It is worth waiting for a view values to be considered before doing this, so students are aware of the power of this simple change. Critically, students can be shown a graph. Here, we set the calculator up to avoid showing the axes. In this way, students focus on the shape only. Again, the graph should only be shown after a lot of data has been considered. I show it only for a couple of seconds and then return to the table of values. The intention is to help get a 'feel' for the function. Then with all this input students will be able to give inputs and outputs, before the function can finally be revealed.

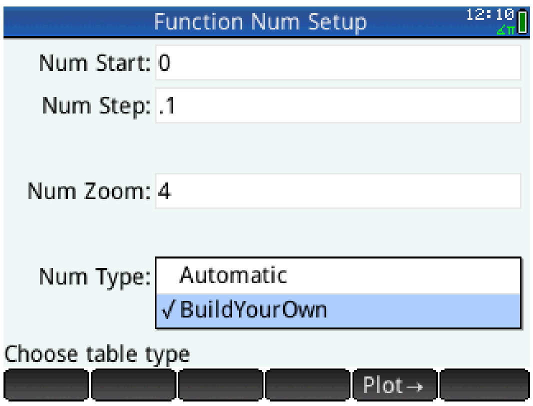


This is how to do it:

First press Apps, choose Function, Reset, OK and Start. Enter a function for your students to guess. (I find a quadratic in the form $(x + a)(x + b)$ works well).



Now click Shift and Plot to enter the plot setup. Click the Page 1/2 softkey, select Axes and click the ✓/CHK softkey to deselect the axes. Click Plot to make sure you can see the graph properly (you may need to use Menu then Zoom to set it up, but quicker to choose a function whose graph you can see!)

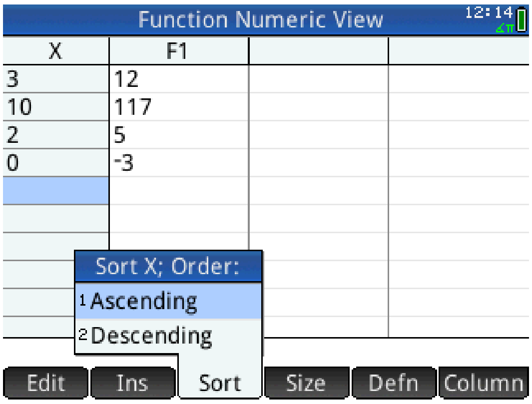


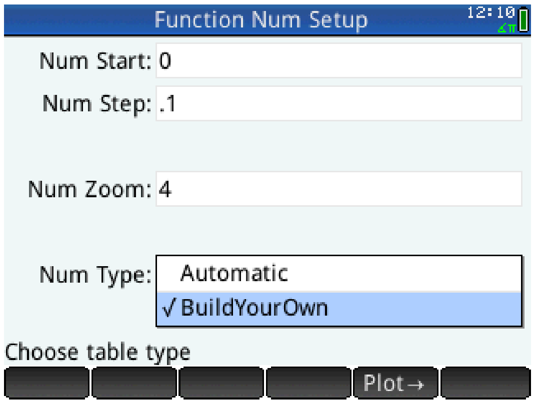
Now click Shift Num to enter the number setup, select Num Type, click the Choose softkey, select BuildYourOwn (and click OK).

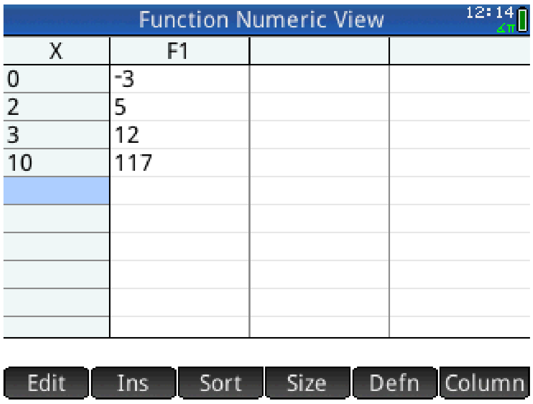
Now click Num and you are ready to start.

Ask participants to give a value for X and guess the value for F1. Make sure no-one actually guesses the function or that will close down everyone else's thinking.

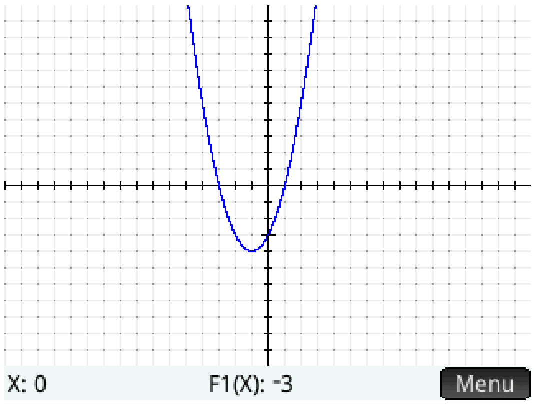
When you have a few values, ask if we could improve the table and they will say (because they always do!) "put them in order", so click the SORT softkey.







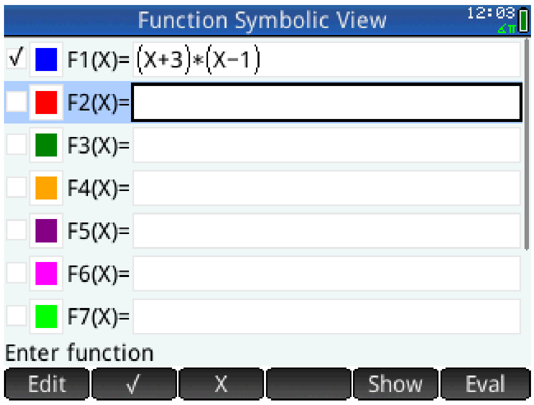
Only when there is enough data and some are struggling, ask if another view would be useful and they'll say "show us a graph". So, click Plot, but only very briefly, click Num again after a couple of seconds. They only need the shape after all!



Now let participants try more new values until they are all confident they have found the function.

As a final view, you can turn the axes back on (Shift/Plot Setup/Page $^1/_2$ /Axes/✓ or CHK)

When most participants are able to give a value for X and its corresponding value of F1, ask them what form they think you put the function in. Finally, the function can be revealed.



It is interesting to discuss how different algebraic forms are better or worse for identifying the graph.

6: Pythagorean Triples

This activity is based on an article by Bill Richardson which is highly recommended and will provide all you need to extend this introduction (last accessed 18.10.13): www.math.wichita.edu/~richardson/pythagoreantriples.html

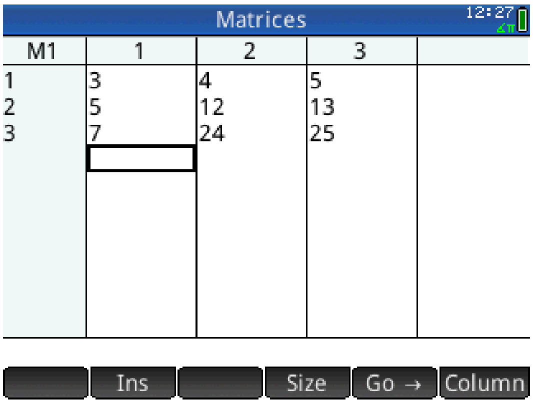
The idea is that if we know the first 4 Pythagorean triples in a sequence connected by some rule, then we can find a generating matrix to produce the others. Students can generate triples using a spreadsheet to find sets in this way. The most well-known would be where the last two values are consecutive integers. Now, students could find an algebraic generator for this set, but this is very hard for some other sets, so see how the matrix method works.
The first 4 triples are:
[3 4 5], [5 12 13], [7 24 25], [9 40 41]
Now we can set up a matrix statement showing the first three triples being generated by the 2nd, 3rd and 4th.

$$\begin{pmatrix} 3 & 4 & 5 \\ 5 & 12 & 13 \\ 7 & 24 & 25 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 5 & 12 & 13 \\ 7 & 24 & 25 \\ 9 & 40 & 41 \end{pmatrix}$$

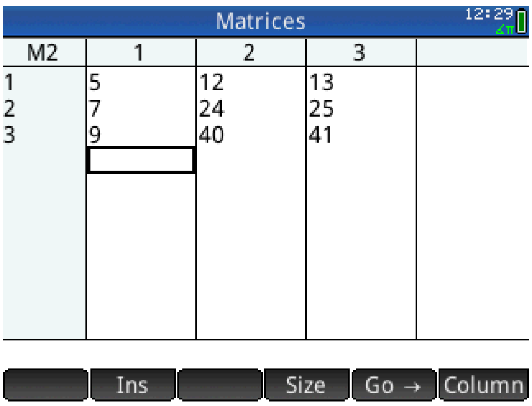
Which we can solve by pre-multiplying by the first matrix i.e.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 3 & 4 & 5 \\ 5 & 12 & 13 \\ 7 & 24 & 25 \end{pmatrix}^{-1}\begin{pmatrix} 5 & 12 & 13 \\ 7 & 24 & 25 \\ 9 & 40 & 41 \end{pmatrix}$$

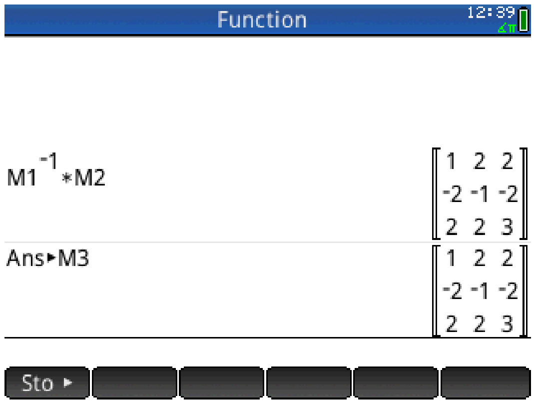
On the calculator we can find the generator matrix using the MATRIX facilities. First we create the two matrices:
Press Shift and Matrix. The cursor should be on the top row for M1. Now press Edit. Enter the values one at a time pressing OK. For the third entry you will need to use the cursor to move to the end of the first row. Press OK. It will now carry on, knowing that the matrix has 3 columns.



The matrix M1 is now defined, so press MATRIX again and repeat for M2



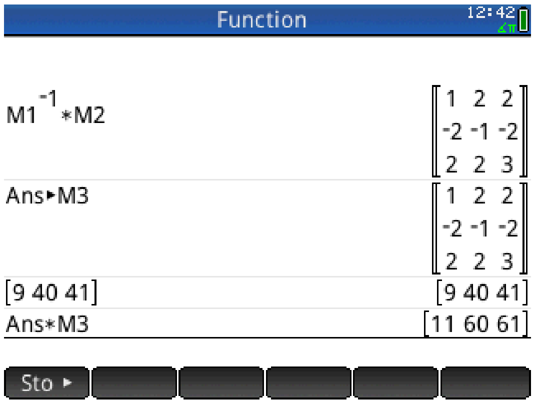
We can now find the generator matrix. So, press HOME and type the statement $M1^{-1}$*M2 and press Enter.
To store this as a new matrix, press Sto then type M3 and press Enter.



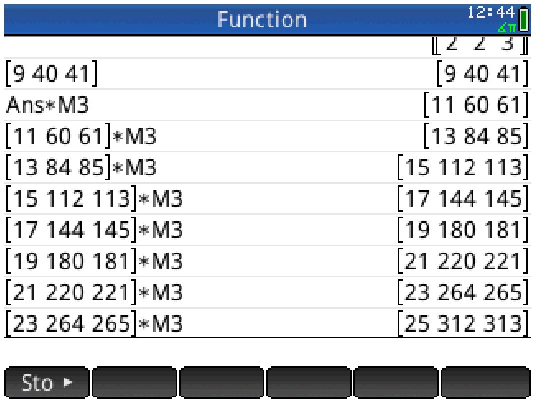
> *Note: To enter matrices directly on the home screen we use the square bracket, one bracket for the matrix, then another for each new row. Everything separated by commas.*

We can now generate triples:
(HP 39gII) Enter the last triple we know as [[9,40,41]]
(HP Prime) Choose square brackets and enter 9 right 40 right 41 Enter

Now press × M3 and Enter
The next triple is [11,60,61]



Now use up arrow and COPY then × M3, then Enter to find the next, and the next, …



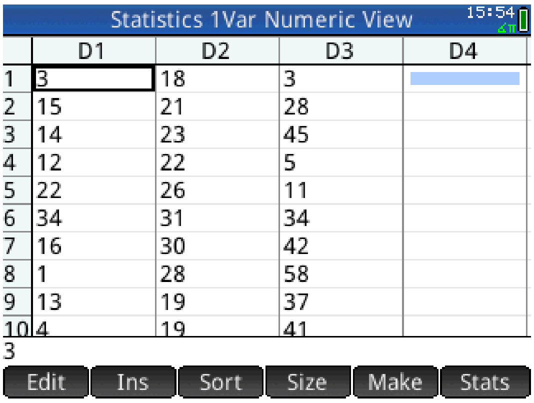
I hope this is sufficiently compelling an idea for students to want to work out how it works. The powerful matrix tools make the mathematics possible. Students need now to go away and work out how matrix multiplication and matrix inverses work. Then they can use this knowledge to work out how the method works and hopefully go on and prove it.

To practice, here are two further sets of triples. Firstly, where the first and last numbers are consecutive odd numbers:
[3 4 5], [15 8 17], [35 12 37], [63 16 65]
Secondly, where the first two numbers are consecutive whole numbers:
[3 4 5], [20 21 29], [119 120 169], [696 697 985]

7: Statistics

A very powerful feature of the graphing calculator is the ability to generate statistical charts and graphs and summary statistics for data sets, very quickly. This allows students to see the multiple representations: the data set, the chart, the summary statistics. They can see the relationships between these representations. If we have single variable data, then we are likely to be comparing two or more data sets. We want to see the differences in distribution and we want to find summary statistics to make comparisons. Box-and-whisker plots are widely seen to be excellent visual tools for comparing single variable data sets. They clearly show the inter-quartile range together with the outlying quartiles and visually showing the average (median), and the range and inter-quartile range. Two such plots side by side allow powerful and immediate inference to be drawn. The HP Prime and HP39gII allow up to three box-and-whisker plots to be drawn side-by-side.

Step-by-step instructions for this work are on the following pages.



Statistics 1Var Numeric View 15:54

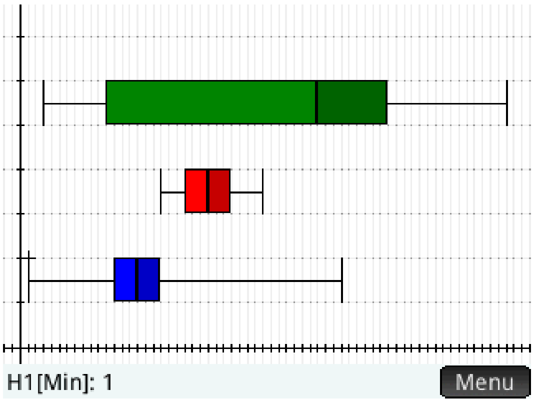
	D1	D2	D3	D4
1	3	18	3	
2	15	21	28	
3	14	23	45	
4	12	22	5	
5	22	26	11	
6	34	31	34	
7	16	30	42	
8	1	28	58	
9	13	19	37	
10	4	19	41	

3

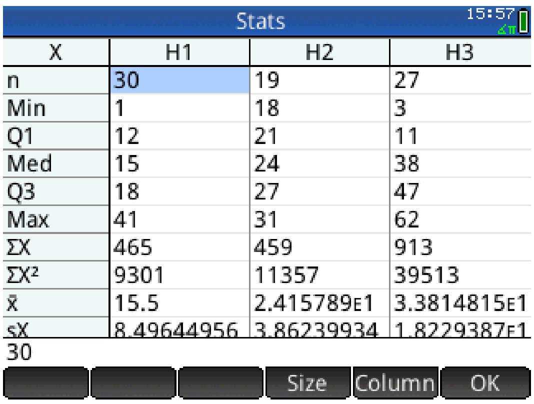
Edit | Ins | Sort | Size | Make | Stats

Students could collect data on (say) the age profile of visitors to three restaurants in town to support an investigation into their marketing requirements.

We can enter the data, draw the plots and see detailed summary statistics:



This is achieved very quickly and now allows a detailed statistical analysis of the data, to support the project at hand.



Stats 15:57

X	H1	H2	H3
n	30	19	27
Min	1	18	3
Q1	12	21	11
Med	15	24	38
Q3	18	27	47
Max	41	31	62
ΣX	465	459	913
ΣX²	9301	11357	39513
x̄	15.5	2.415789E1	3.3814815E1
sX	8.49644956	3.86239934	1.8229387E1

30

Size | Column | OK

Similarly 2 variable data can be handled equally efficiently, generating scatter plots, with regression and correlations calculated and different regression analysis available.

Step-by-step instructions start on page 27.

Drawing Three Side-by-Side Box and Whisker Plots

Choose the Statistics Applet:
Press Apps and use the arrow keys to find Statistics 1Var
Use the SoftKeys for Reset and OK and Start

Enter your Data:
Type data in column D1 and press Enter repeat until finished
(Use the right and left arrow keys to move between the lists)

- Under D1 enter the data for the first set
- Under D2 enter the data for the second set
- Under D3 enter the data for the second set

Choose the Plots:
Press the Symb key
Plot H1 will already be set to D1
Press the down arrow to set plot H2 use the SoftKey then 2 then OK
Repeat for H3.

For each plot select the Plot type (a drop down on HP Prime) and press CHOOS (HP39gII) and select BoxWhisker and OK

Now press the Plot key and press Menu/Zoom/Autoscale to optimise the view.

Get the Statistics:
In the Plot view, press Menu/Trace and use the cursor keys to show on-screen summary statistics.

- Minimum value
- Lower Quartile (Q1)
- Median
- Upper Quartile (Q3)
- Maximum value

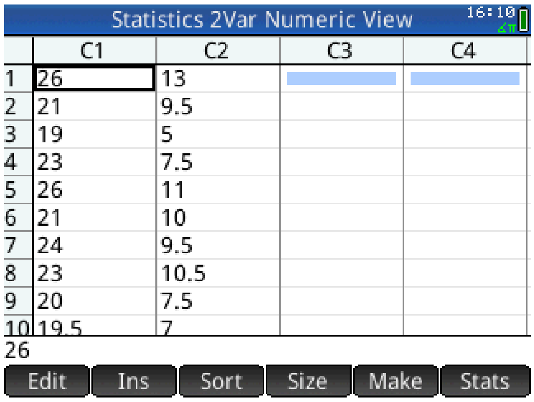
Students can work out and write down the interquartile range.
Remembering: Interquartile range = upper quartile – lower quartile

Find the Statistics for the Data
Press the Num key and press Stats for detailed summary statistics.
Press SoftKey OK when finished.

Two Variable Statistics: Correlation and Regression

Choose the Statistics Applet:

Press Apps and use the arrow keys to find Statistics 2VAR
Use the SoftKeys for Reset and OK and Start

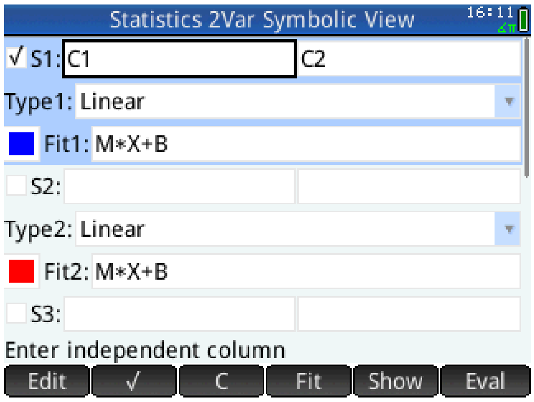


Enter your Data:

Type data in column C1 and press Enter repeat until finished
(Use the right and left arrow keys to move between the lists)

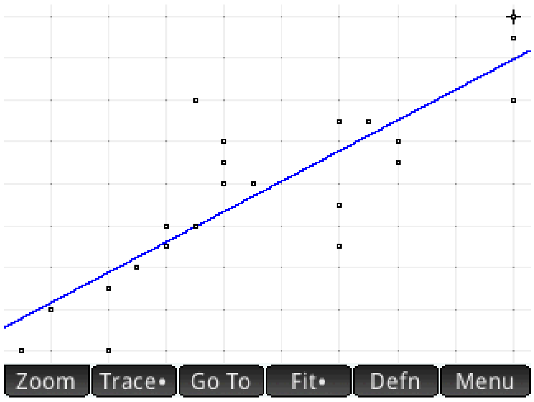
- Under C1 enter the data for the first set
- Under C2 enter the data for the second set

Make sure both lists are the same length.

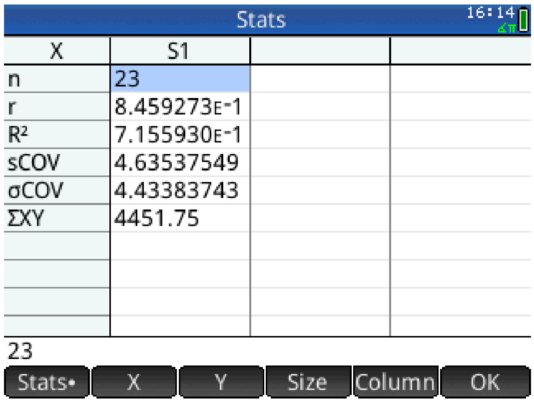


Choose the Plots:

Press the Symb key
Plot S1 should already be set to lists C1 and C2
Now press the Plot key
Press Menu Zoom and use the arrow keys to choose Autoscale

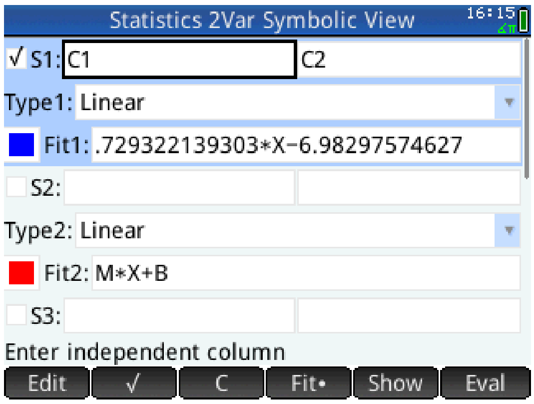


Press Menu Fit to show the line of best fit. Press again to remove it.



Find the Statistics for the Data:

Press the Num key and SoftKey Stats to show detailed summary statistics.
Press SoftKey OK when finished.
(HP39gII) Use the down arrow to see all the statistics.
Press the Symb key and the regression line will now be showing as Fit1
Press SoftKey Show to see it separately.



8: Data Streaming

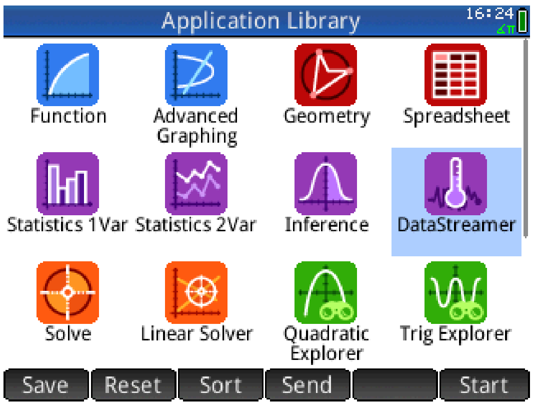
Setting Up

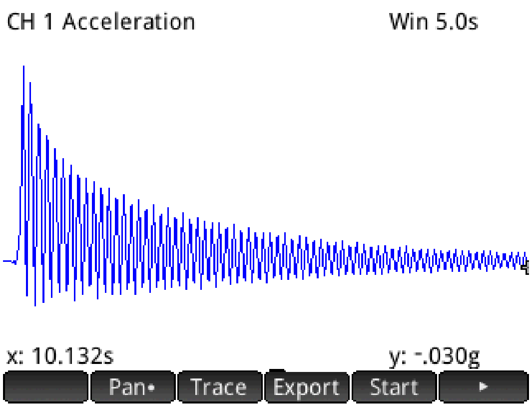
You will need an HP Prime or an HP39gII and a StreamSmart 410 interface unit.

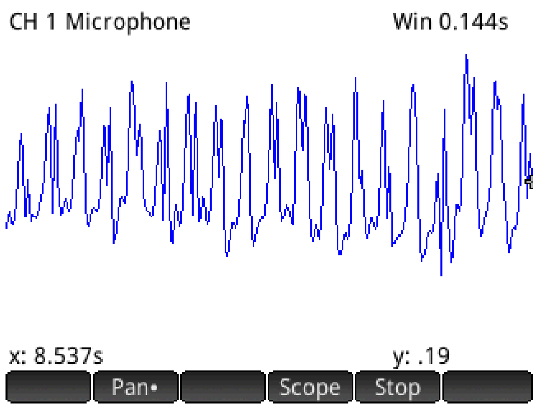
You will also need suitable probes for the experiment.

Check that the probe you are using is connected to the CH1 input on the StreamSmart unit and that this is connected via it's attached lead to the calculator. Also check that the StreamSmart unit is switched on and the green light is showing that the battery is OK.

On the calculator, press Apps an use the arrow keys to find DataStreamer. When it is highlighted press the SoftKeys Reset then OK then Start.







There are a variety of probes which can be used. The first example above show the output when an acceleration probe is attached to end of a steel rule held firmly on the edge of a table and 'twanged'. The second example is the output from blowing into a microphone.

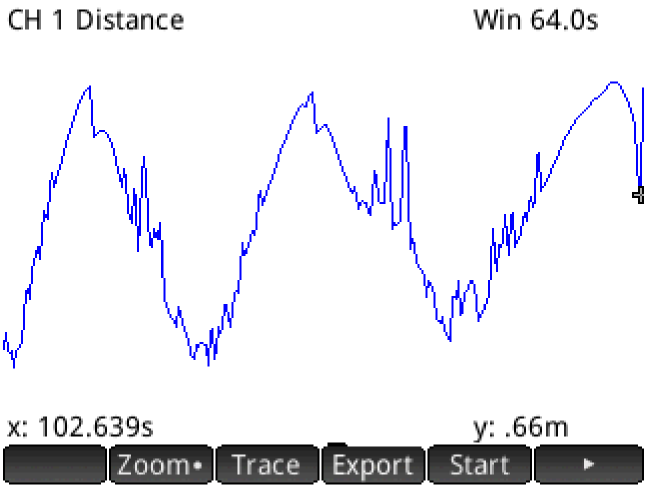
A Distance Probe can be used for matching distance time graphs. The following activity uses this feature. Also, mechanics experiments can be measured or pendula for simple harmonic motion etc. A temperature probe can be used for analysing cooling (or heating) curves.

Walking the Line

Follow the set up instructions above with the Distance Probe attached to the StreamSmart unit.

In this activity, students will walk along a line closer to and further away from the distance probe.

Put the probe on a table, facing a space at least 1 metre wide and 3 metres long. The centre of a classroom is good if the desks are moved apart a little. Find a volunteer and ask them to stand facing the probe, about 2 metres away.



We want to set up the screen so that we can see the motion.

Press the SoftKey Pan and it will change to Zoom. Press it again and it changes back. We will use Pan and Zoom to set up the screen. Pan moves the image up and down. Zoom makes the image expand in scale.

Choose Zoom and press the up arrow about three times to increase the scale.

Then choose Pan and put your hand in front of the sensor to measure near to zero metres. Now use the up arrow to move the line to just above the numbers on screen. Take your hand away to measure the person 2 metres away again.

So the 2 metre point is about 2/3 of the way up the screen (so full screen will show 3 metres) and the bottom of the screen shows zero metres (i.e. with the hand in front).

This is a little fiddly the first time, but you will get better with practice.

The Activity

You will need a collection of drawings of standard distance time graphs from a text book. The aim is for students to match their motion to the graph.

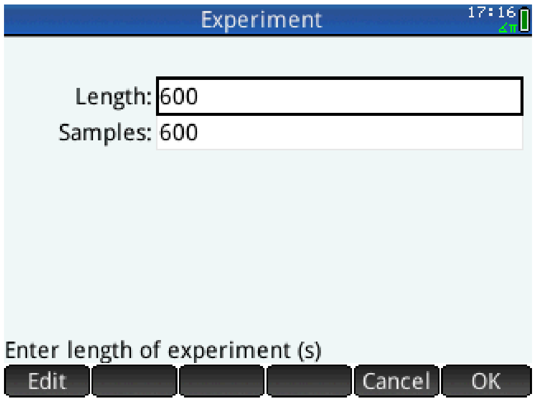
The student on the line should move backwards and forwards (always facing the probe) until they have created the required graph.

Teachers can be inventive with which graphs they use!

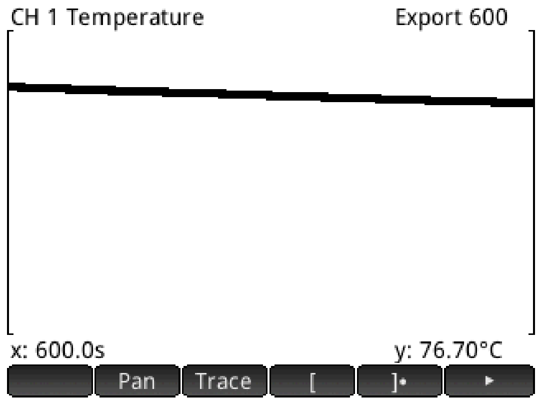
The author has prepared a presentation sequence with a variety of graphs which can be printed or displayed to support this activity. It is available on request.

Cooling Curves

Set up the StreamSmart Unit with a temperature probe (a thermometer). Start the DataStreamer App as before. For this experiment we will set up the length and the number of samples to be collected in advance, this way, the screen will show the whole experiment as it unfolds.

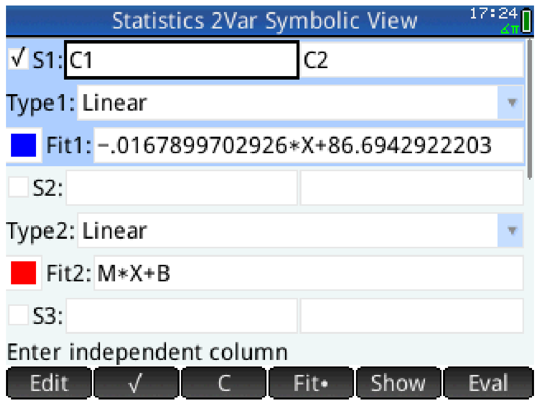


It is important that the thermometer has reached the temperature of the materials being cooled before the experiment starts. You can check by pressing Start in the DataStreamer App and monitoring the temperature to see it is not still rising. When it is not rising, work quickly to start the experiment.

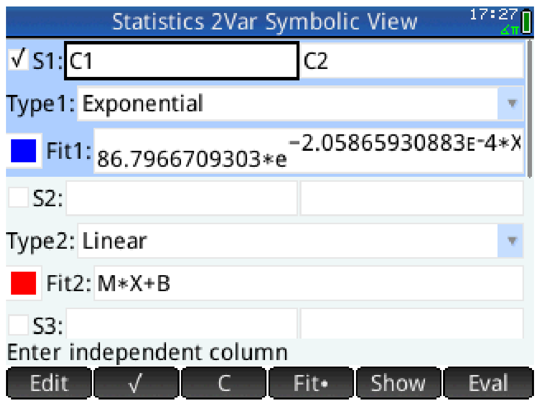


Press the View key. Enter 600 for the experiment length press OK, then 600 for the number of samples and press OK. Now, when everything is ready and set up to start the experiment, press OK again and the data logging should start and you should see a graph begin to appear on screen.

The experiment can be used to keep the students' focus on how we make a mathematical model to describe the rate of cooling, so it is appropriate to take data samples at intervals throughout the experiment. The details of this experiment have been published by the author as Modelling, Functions and Estimation: A Pizza Problem [1].



When the experiment is finished, the data can be exported to the two variable statistics applet. Press the Export SoftKey then the right Arrow SoftKey, then OK. This will automatically launch the Statistics 2Var App. You can press the Plot key to see your graph again. This cooling 'curve' looks remarkably linear. (The Experiment was done with freshly boiled water in an open cup). If we press the Symb key, we can see a best fit linear model. Then we can select a different model to see if we can improve the quality of the fit. Here, we have chosen an exponential model.



1: Olley, C. (2011) Modelling, Functions and Estimation: A Pizza Problem In, Oldknow, Adrian and Knights, Carol (eds.) *Mathematics Education with Digital Technology.* , Continuum, London.

Notes

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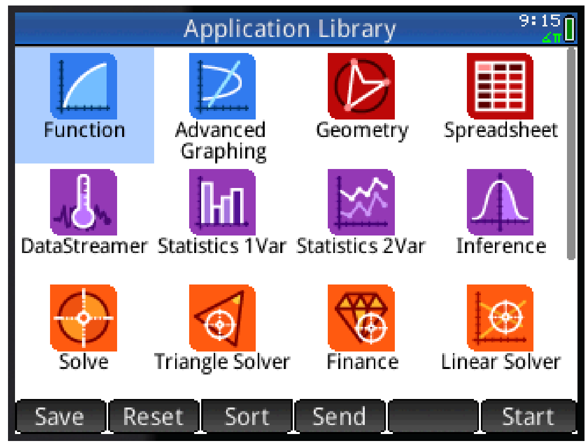
HP Prime Advanced Graphing Calculator

HP Prime is the latest advanced graphing calculator from Hewlett-Packard. It incorporates a full-color, multi-touch screen and comes pre-loaded with 18 HP Apps. HP Prime also has a Home view, with a history of your numerical calculations, as well as a CAS view, with a history of your symbolic computations.

HP Prime has three design principles:

- Provide a high degree of mathematical fidelity across multiple mathematical representations
- Give students the interactive experiences with technology that they have come to expect
- Deliver a simple and seamless experience for mathematical exploration and problem-solving

HP Apps are designed to explore mathematical topics or solve problems. All HP Apps have a similar structure, with numeric, graphic and symbolic views to make them easy to learn and easy to use. Press Apps Info to see the App Library. Drag with your finger to browse the library, then tap the icon of the app you want to use.



The HP Apps are color-coded for easy identification:

- 4 Solver Apps (orange) for solving specific types of problems (triangles, finance, linear systems) and a general solver for equations and non-linear systems
- 4 Statistics Apps (purple) for descriptive and inferential statistics and data collection
- 4 graphing apps (blue) to explore graphs and tables of values
- 3 Explorer Apps (green) for investigating the relationship between the parameters of a function and the shape of the function's graph

In addition to these apps, there are four more apps of particular interest:

- The Advanced Graphing App allows you to plot and explore the graphs of functions and relations in *x* and *y*, including implicit relations, inequalities, and conic sections
- The Geometry App gives you the features of a dynamic geometry application, but with a Symbolic view for defining geometric objects exactly using the CAS
- The Spreadsheet App gives you the features of a spreadsheet with the power of a CAS
- The Data Streamer app lets you collect data from sensors quickly and easily

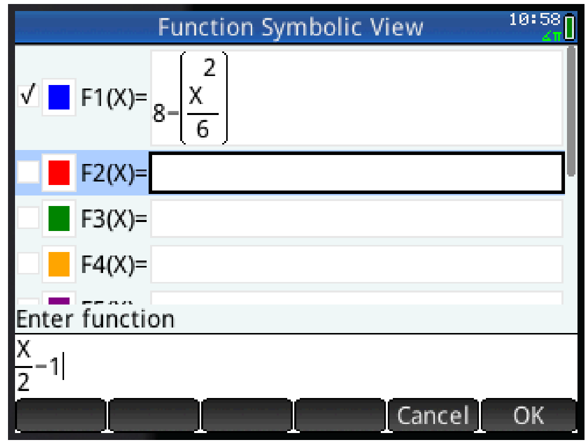
In this brief introduction to HP Prime, we will examine these four apps, but first we will look at the Function App.

The Function App

The Function App gives you all the tools you need to explore the properties of functions, including plotting their graphs, creating tables of values, and finding roots, critical points, etc.

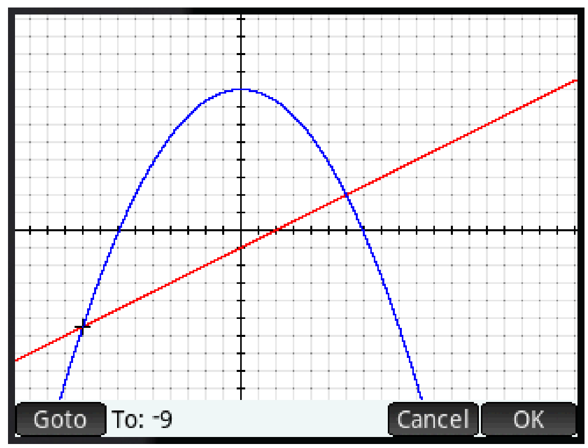
Tap on the Function App icon and it opens in the Symbolic view. Enter a function using the 2-D textbook format editor.

Choose a color for each of your graphs from the color picker.



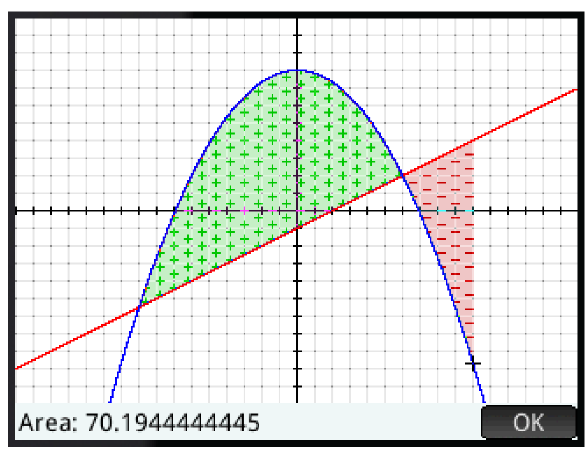
Press Plot Setup to see the graphs of your functions.

In the Plot view, tap Fcn and then tap an option. We will use the Signed Area function.



You can find the signed area between the two graphs. Just tap near the starting point and then use the arrow keys to get the exact point you want.

Now tap near the end point and again use the arrow keys to get the exact value. With the touch display, the experience is more responsive and interactive.



The color display shows you which regions have positive area and which have negative area.

You can use your finger to drag, which scrolls you through the window. You can also press + and − to zoom in and out on the cursor. No more going back and forth between the plot and the window settings!

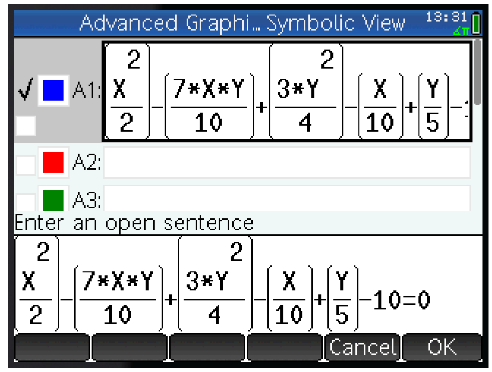
The Advanced Graphing App

The Advanced Graphing App is designed to plot the graphs of conic sections, polynomials, inequalities – virtually any mathematical open sentence in X and Y.

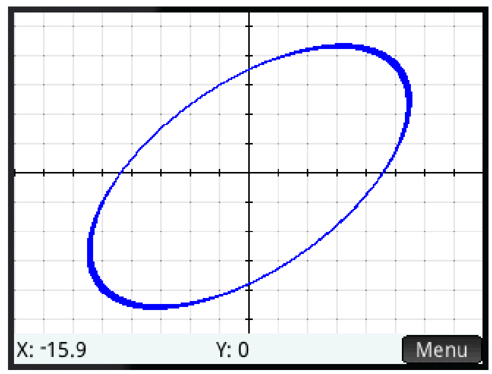
Tap the icon to see the Symbolic View and enter an open sentence in X and Y using the 2-D editor.

Here we enter a rotated conic in general form; in this case, an ellipse.

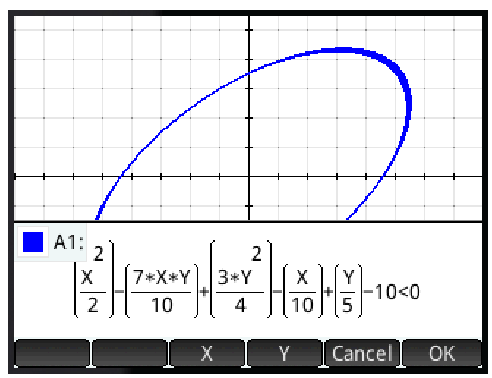
Use the color picker to choose a color for your graph.

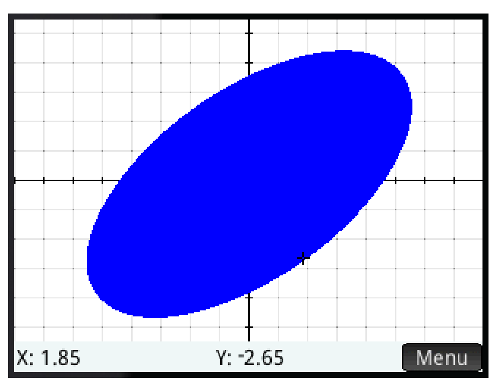


Now press Plot Setup to see the graph of this rotated conic section. Drag your finger to scroll the window, or use + and − to zoom in and out. The touch interface makes it easy to adjust the graph without leaving the Plot view.



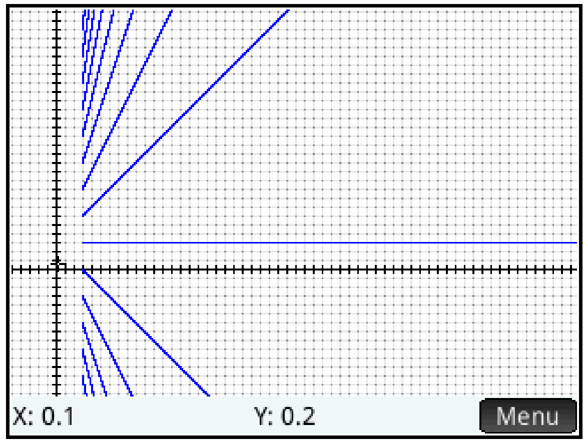
While in the Plot view, tap Menu and Defn to view and edit the equation-again without leaving the Plot view. Change the equal sign to < and then tap OK to see the new graph.





The Advanced Graphing App can handle all of the following with ease:

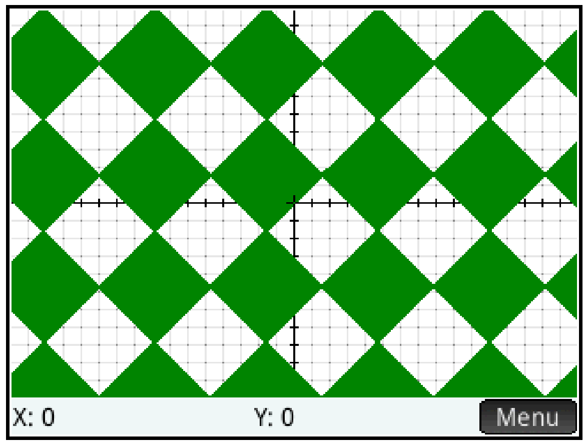
- Conic sections (rotated ones, too)
- Polynomials in standard or general form
- Inequalities (not just linear)
- Functions



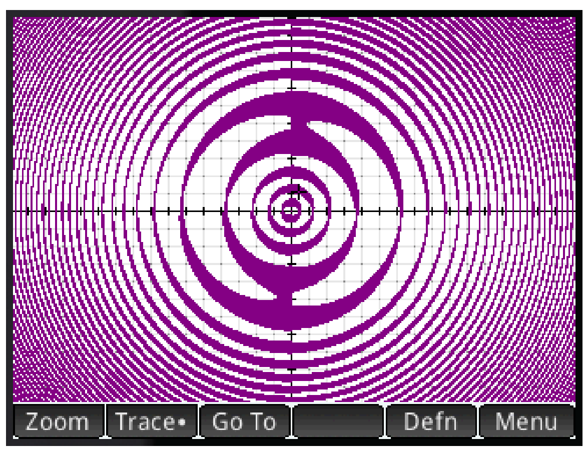
y Mod $x = 3$

In addition, it can handle graphs you have not thought about, like the figures to the right.

There are new teaching opportunities for you and new learning opportunities for your students with the Advanced Graphing App!



Sin(x)<Sin(y)

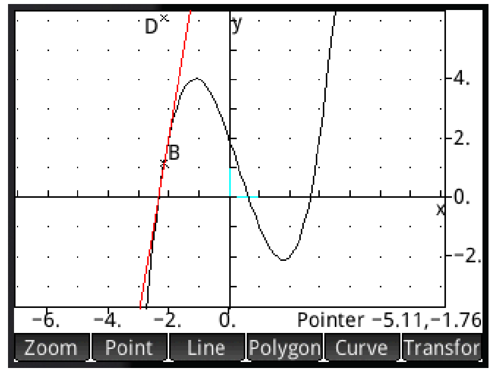


$$Sin\left(\left(\sqrt{x^2+y^2}-5\right)^2\right) > Sin\left(Tan^{-1}\left(\frac{y}{x}\right)\right)$$

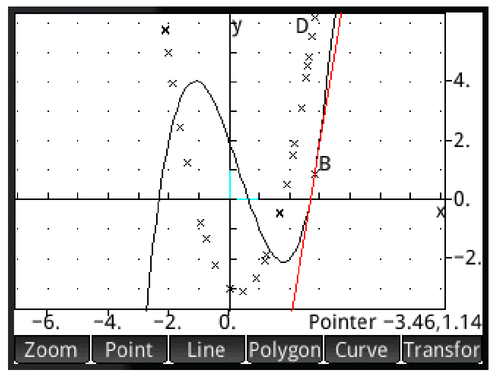
The Geometry App

The Geometry App is a dynamic geometry application that allows you to create geometric constructions and explore their properties interactively.

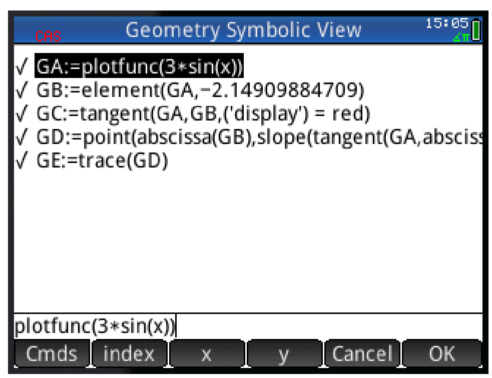
In this construction, point B is the point of tangency for the red tangent line. The graph in black is a cubic function.



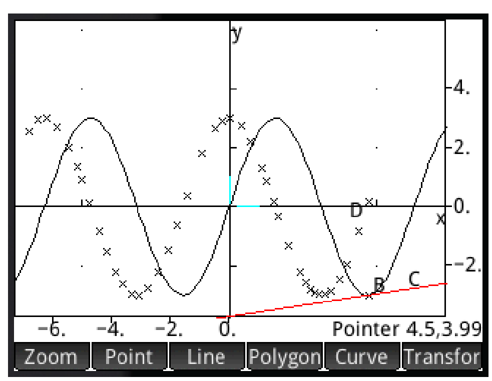
Point D has the same x-coordinate as point B but its y-coordinate is equal to the slope of the tangent line. As point B moves along the curve, point D traces out the derivative of the curve.



The Symbolic view of the app shows the definition of each object in the Plot view. You can edit these definitions directly or create new objects in this view. Check the ones you want to show in the Plot view and uncheck the ones you want to hide. Change the graph being plotted in GA to y=3sin(x). Point D still traces out the derivative.

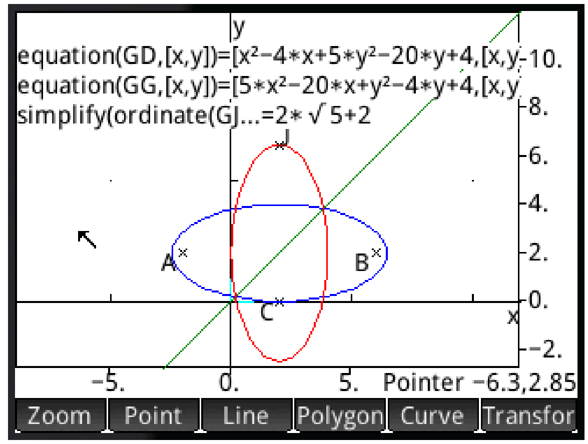


This one construction can now help your students visualize the derivative of <u>any</u> function!



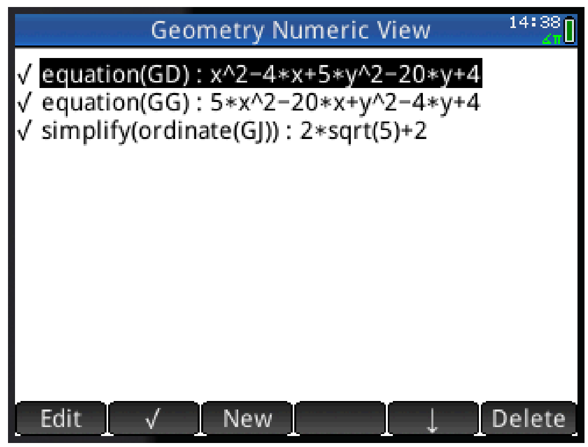
Save this construction with a name you'll remember and use the Geometry App to make other constructions.

In this construction, we have a blue ellipse defined by the focal points A and B, and a point C on the ellipse.

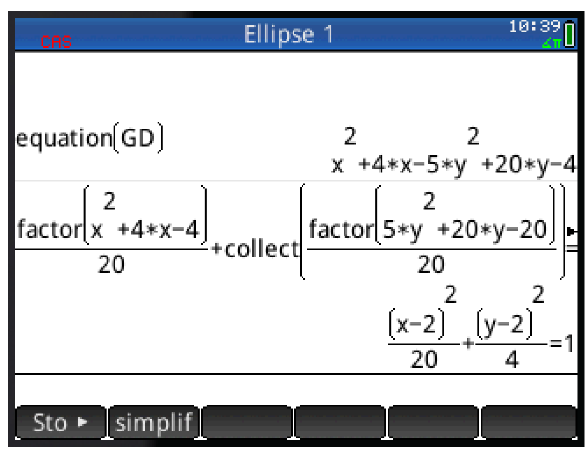


The red ellipse is the reflection of the blue one over the line y=x. They are inverses of each other. Note the equations of the two ellipses and how the roles of *x* and *y* are interchanged. Since the points A, B, and C were defined exactly in the Symbolic view, the equations are exact as well. Also, the y-coordinate of point J is shown to emphasize the exact nature of the result.

The Numeric view of the Geometry app is where you go to make measurements or create calculations based on measurements. Check the ones you want displayed in the Plot view.



Here the CAS is used to retrieve the equation of the blue ellipse and put it into a more familiar form.

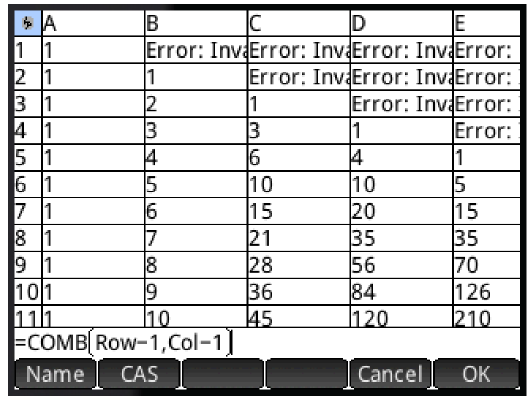


Note the length of the major semi-axis is $\sqrt{20} = 2\sqrt{5}$ and the center is at (2,2). So the point J on the reflected ellipse should have a y-coordinate of $2 + 2\sqrt{5}$, which it does exactly as shown in the figure above.

The integration of the CAS and dynamic geometry gives you an exact analytical geometry app that is dynamic as well.

The Spreadsheet App

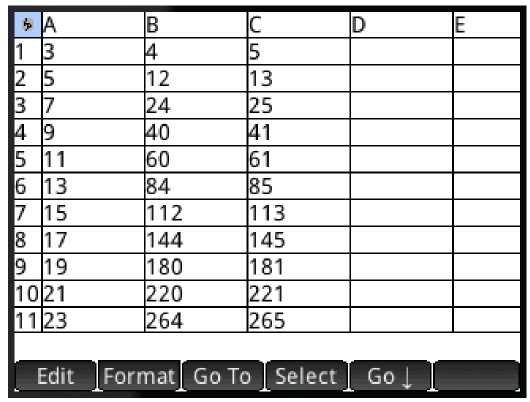
The Spreadsheet App gives you the most common features you expect in a spreadsheet. But with HP Prime, you also get the power of a CAS integrated with the spreadsheet.



	A	B	C	D	E
1	1	Error: Inv	Error: Inv	Error: Inv	Error:
2	1	1	Error: Inv	Error: Inv	Error:
3	1	2	1	Error: Inv	Error:
4	1	3	3	1	Error:
5	1	4	6	4	1
6	1	5	10	10	5
7	1	6	15	20	15
8	1	7	21	35	35
9	1	8	28	56	70
10	1	9	36	84	126
11	1	10	45	120	210

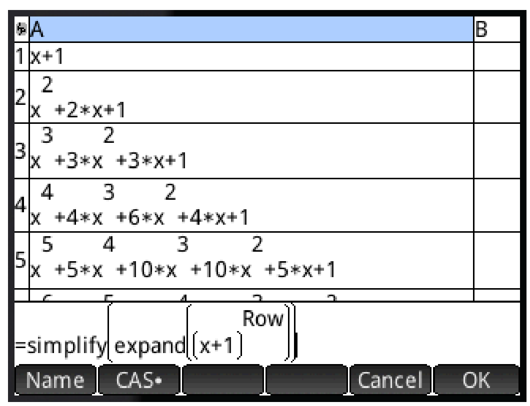
You can define the entire spreadsheet with a single command. Here we produced Pascal's Triangle using just the nCr command (COMB) and the Row and Col variables.

Or how about a handy reference table of the first thousand or so Pythagorean triplets? It's easy!



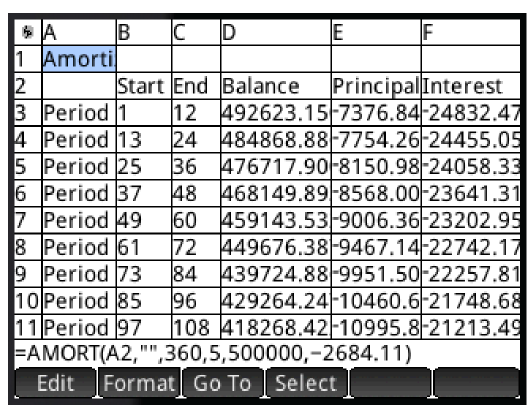
	A	B	C	D	E
1	3	4	5		
2	5	12	13		
3	7	24	25		
4	9	40	41		
5	11	60	61		
6	13	84	85		
7	15	112	113		
8	17	144	145		
9	19	180	181		
10	21	220	221		
11	23	264	265		

Here we used the CAS to define one column of the spreadsheet to expand the binomial $x+1$ to various integer powers. Note the expression editor shows a CAS button. When active, the CAS is used to evaluate the expression. When it is not active, expressions are evaluated to numerical results, as in the previous examples. Imagine all the patterns you and your students can explore in symbolic expressions!



	A	B
1	$x+1$	
2	$x^2+2*x+1$	
3	$x^3+3*x^2+3*x+1$	
4	$x^4+4*x^3+6*x^2+4*x+1$	
5	$x^5+5*x^4+10*x^3+10*x^2+5*x+1$	

The Spreadsheet App can also handle commands with multiple outputs, such as summary statistics. Here, the AMORT command was used to display an amortization table for a 30-year, \$500,000 loan at 5% interest with monthly payments of \$2684.11.



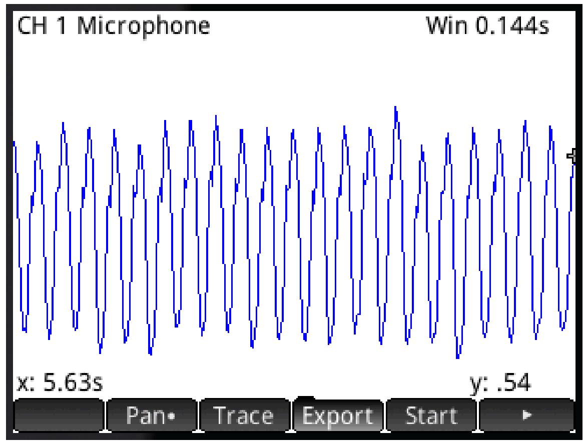
	A	B	C	D	E	F
1	Amorti					
2		Start	End	Balance	Principal	Interest
3	Period	1	12	492623.15	-7376.84	-24832.47
4	Period	13	24	484868.88	-7754.26	-24455.05
5	Period	25	36	476717.90	-8150.98	-24058.33
6	Period	37	48	468149.89	-8568.00	-23641.31
7	Period	49	60	459143.53	-9006.36	-23202.95
8	Period	61	72	449676.38	-9467.14	-22742.17
9	Period	73	84	439724.88	-9951.50	-22257.81
10	Period	85	96	429264.24	-10460.6	-21748.68
11	Period	97	108	418268.42	-10995.8	-21213.49

The Data Streamer App

The Data Streamer App works with the HP StreamSmart 410 and up to four Fourier® sensors to collect data in real time. The final data set is sent to one of the statistics apps for analysis.

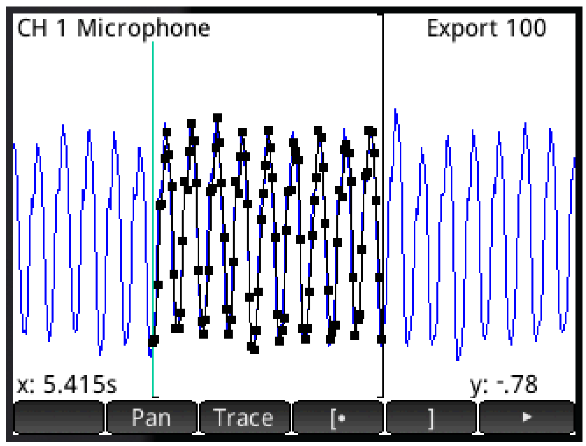
Just plug the microphone sensor into the StreamSmart 410 and start the Data Streamer App. The microphone is automatically identified. Just tap Start to see your own voice in real time. Zoom in/out while data is still streaming.

Here, all the data you see was collected in 0.144 seconds-roughly 1/7 of a second!

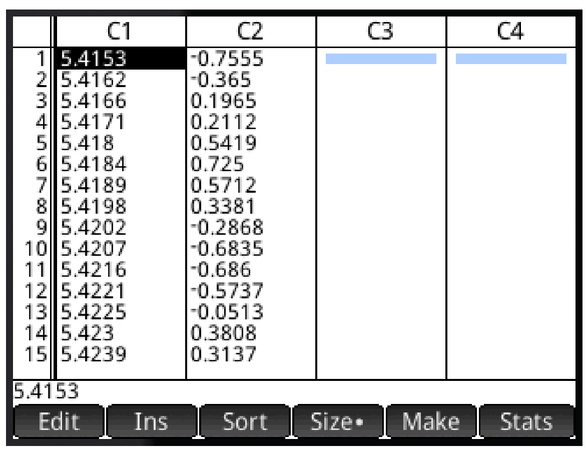


Tap Export to select a final data set. You can crop data from the left and right, as well as thin your data set to get just the data you want.

Once you have selected a final data set, send your data to one of the Statistics Apps for analysis.



Here we see the same data numerically in the Numeric view of the Statistics 2Var App. From here, you can make a scatter plot of the data and create a trigonometric fit.

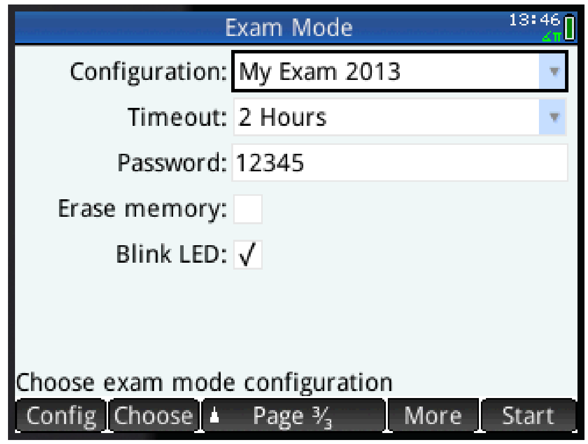


	C1	C2	C3	C4
1	5.4153	-0.7555		
2	5.4162	-0.365		
3	5.4166	0.1965		
4	5.4171	0.2112		
5	5.418	0.5419		
6	5.4184	0.725		
7	5.4189	0.5712		
8	5.4198	0.3381		
9	5.4202	-0.2868		
10	5.4207	-0.6835		
11	5.4216	-0.686		
12	5.4221	-0.5737		
13	5.4225	-0.0513		
14	5.423	0.3808		
15	5.4239	0.3137		

Examination Mode

HP Prime can be configured and locked for an examination. The machine will remain locked for the pre-set time period and secured with a password. LED lights at the top of the unit will flash to show that it is in examination mode.

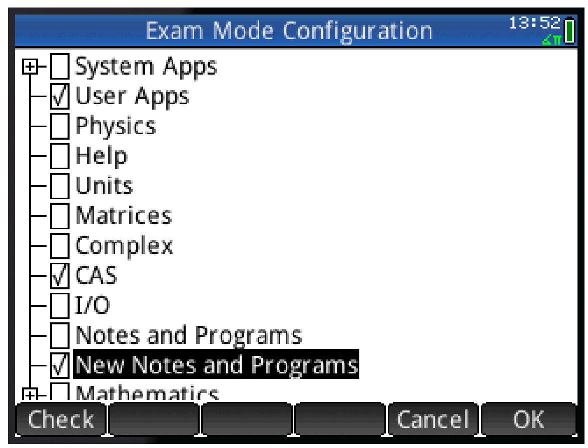
Go to the Exam Mode interface, give your configuration a name, set a time period, and give it a password. You can also check a box to erase memory when examination mode starts, and check another box to make the LED lights blink while in examination mode.



Tap Config to select the features you want to disable.

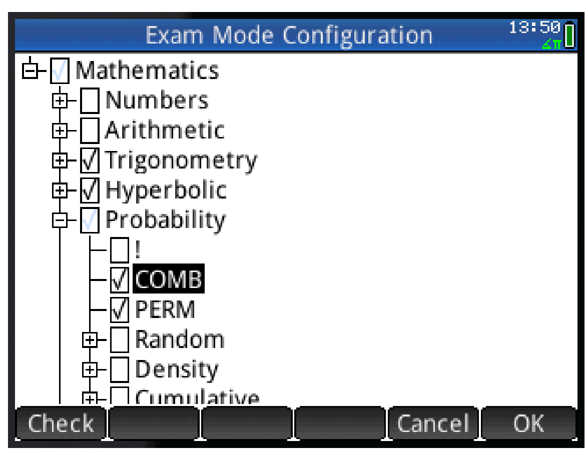
Here we have disabled all saved apps, the whole CAS, and new notes and user programs.

You can block individual apps, all matrix operations, units, even the Help system.



In the Mathematics menu, you can disable commands by category or individually.

Create as many custom configurations as you like and save them with names you will remember. You can edit any saved configuration as well. Start the examination mode on the HP Prime or send it to another HP Prime via USB. Once examination mode has started, the LED lights will blink to show that the configuration is in effect.



Introduction

HP Prime is the latest advanced graphing calculator from Hewlett-Packard. It incorporates a full-color, multi-touch screen and comes pre-loaded with 18 HP Apps. HP Prime has a Home view, with a history of your numerical calculations, as well as a CAS view, with a history of your symbolic computations.

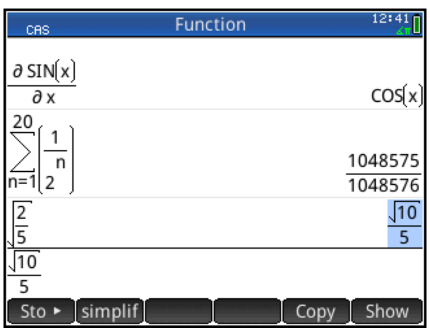
HP Prime has three design principles:

- Provide a high degree of mathematical fidelity across multiple mathematical representations
- Give students the touch-active experiences with mathematics that they have come to expect from display technologies
- Deliver a simple and seamless experience for mathematical exploration and problem-solving

Home and CAS

With HP Prime, you can choose whether to operate numerically in the Home view or symbolically using the CAS in the CAS view. For example, press Settings and enter $\sqrt{8}$ in the Home view to see 2.828… or press CAS Settings and enter the same expression in the CAS view to see $2\sqrt{2}$.

CAS View



Calculation History

- Tap to select an entry
- Tap twice to copy it to the command line

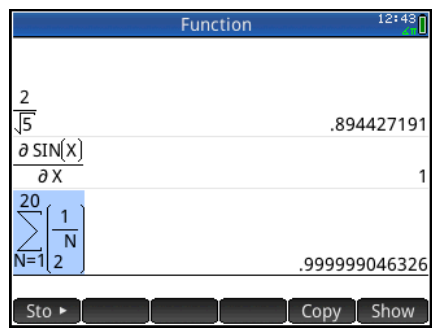
Command Line Editor

- 2D textbook format

Menu Buttons

- Tap to activate
- Context-Sensitive

Home View



Things you can do in both CAS and Home views:

- Tap an item to select it or tap twice to copy it to the command line editor
- Tap and drag up or down to scroll through the history of calculations
- Press Menu Paste to retrieve a previous entry or result from the other view
- Press the Toolbox key (Mem B) to see the Math and CAS menus as well as the Catalog
- Press Units C to open a menu of easy-to-use templates
- Press Esc Clear to exit these menus without making a selection
- Tap Sto ▸, Copy, and Show menu buttons once to activate

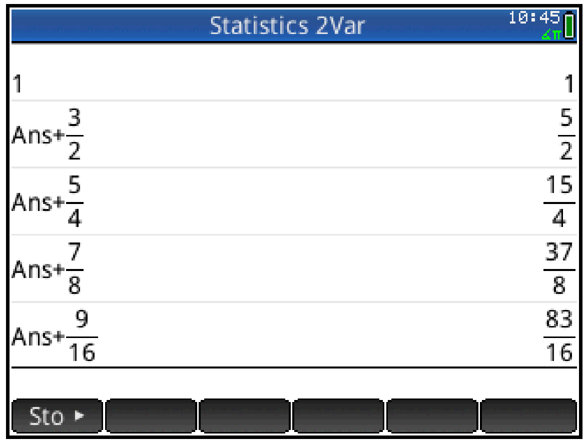
RPN Note: HP Prime supports RPN entry. In Home Settings (Shift Settings), navigate to **Entry** and select RPN. These workshop materials use the default Textbook setting.

Example: Infinite Series

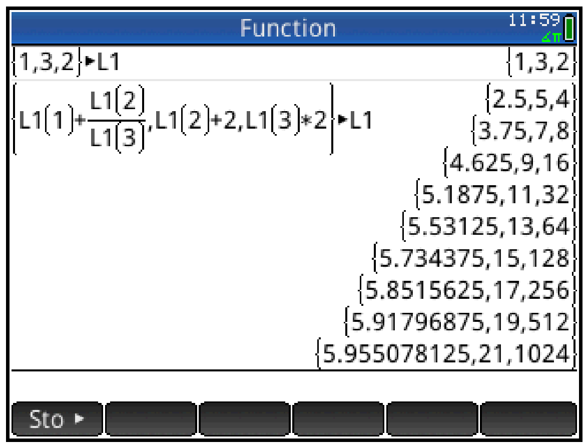
Suppose we wish to explore the series $1+\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\frac{9}{16}...$

To begin with, we can compute the first few partial sums.

1. Press Settings to enter the Home view.
2. Type 1 and press Enter
3. Press + 3 ÷ 2 Enter
4. Press a b/c to cycle through its decimal, proper fraction, and improper fraction forms
5. Continue as shown to the right until you have the first 5 partial sums



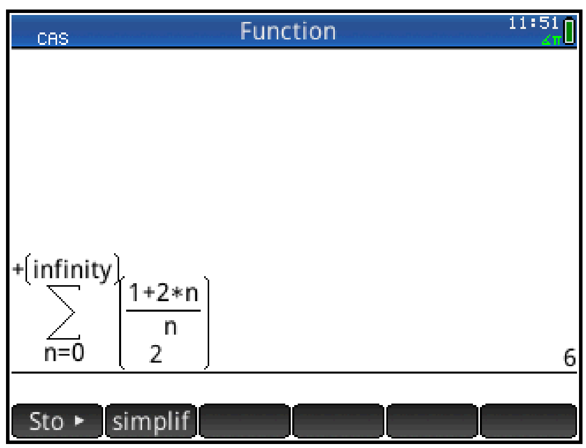
Another approach is to use lists. In the figure to the right, the List L1 is first defined to contain the 1st partial sum (1), followed by the numerator and denominator of the following term. The second line adds the next term to the sum, then adds 2 to the numerator and multiplies the denominator by 2, and stores the result back into list L1. From there, just pressing Enter generates the partial sums.



To use the CAS, we observe that the sum can be rewritten symbolically as

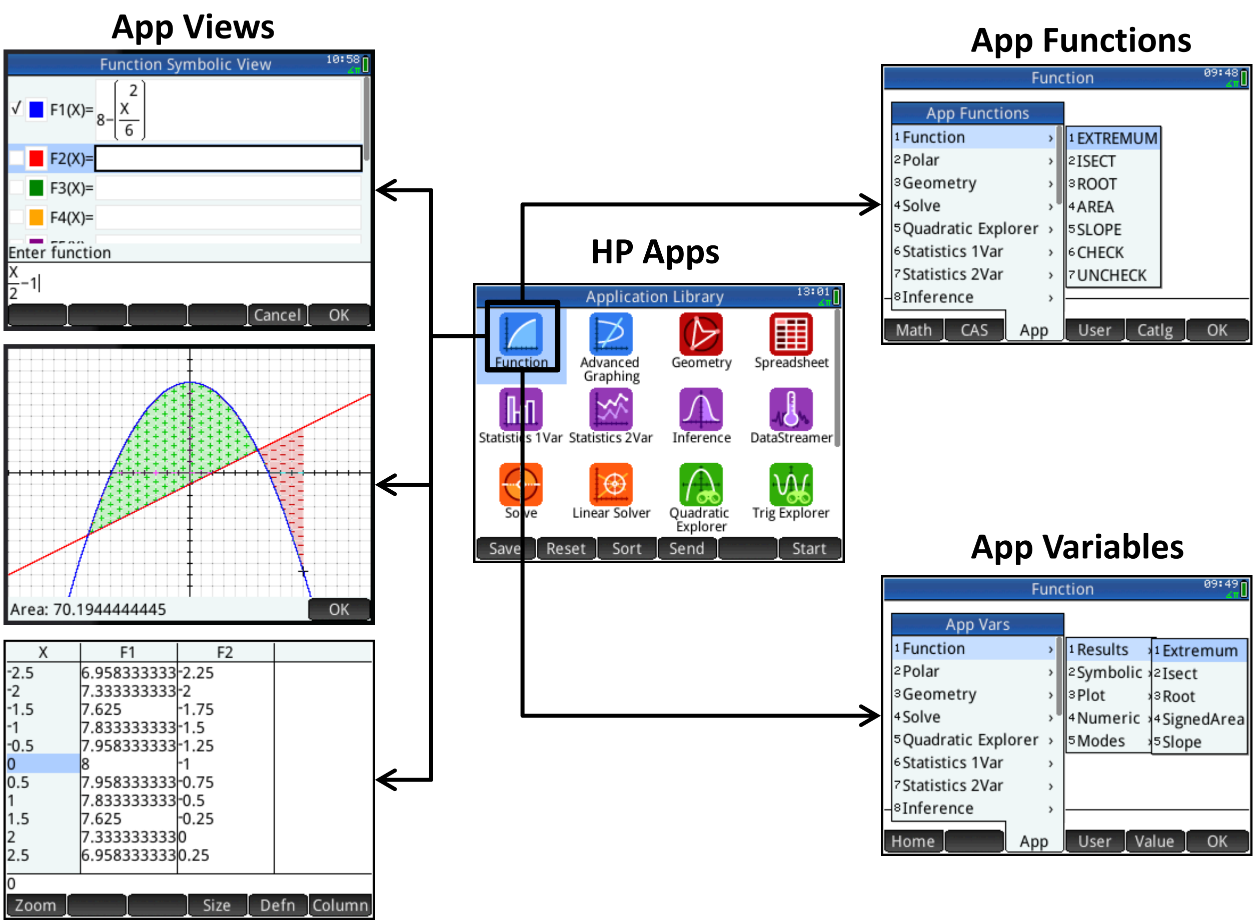
$$1+\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\frac{9}{16}...=\sum_{n=0}^{\infty}\frac{1+2n}{2^n}$$

1. Press CAS Settings to open the CAS view.
2. Press Units to open the template menu and select the summation template
3. To enter n=0, press ALPHA Shift ( ) ▶ 0
4. To enter +∞ at the top, tap on the top and press +, then Shift 9 and tap on ∞
5. Tap on the right template box and enter the rational expression.
6. When you are done, press Enter



HP Apps

HP Apps are designed to explore mathematical topics or solve problems. All HP Apps have a similar structure, with numeric, graphic and symbolic views to make them easy to learn and easy to use. Fill the app with data while you work, and save it with a name you'll remember. Then reset the app and use it for something else. You can come back to your saved app anytime-even send it to your colleagues! HP Apps have app functions as well as app variables; you can use them while in the app, or from the CAS view, Home view, or in programs.

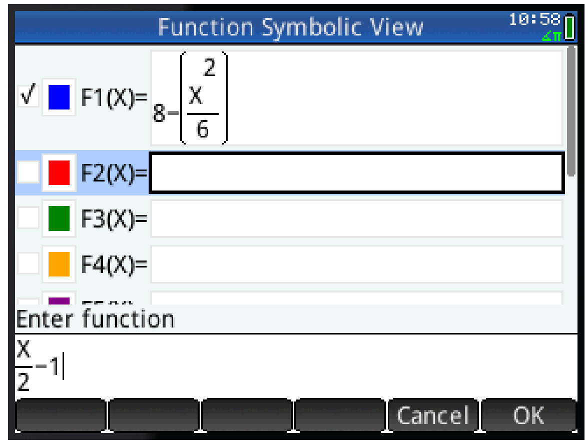


Press Apps Info to see the App Library. Drag with your finger to browse the library, then tap the icon of the app you want to use. The HP Apps are color-coded for easy identification:

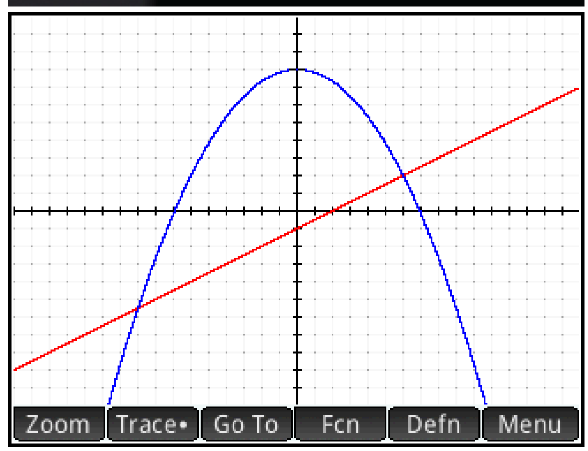
- 5 graphing apps (blue) to explore graphs –including the new Advanced Graphing App!
- 2 Special apps (red): the Geometry app and the Spreadsheet app
- 4 Statistics Apps (purple) for descriptive and inferential statistics and data collection
- 4 Solver Apps (orange) for solving specific types of problems (triangles, finance, etc.)
- 3 Explorer Apps (green) for investigating a function's equation and its graph

The Function App

The Function App gives you all the tools you need to explore the properties of functions, including plotting their graphs, creating tables of values, and finding roots, critical points, etc.

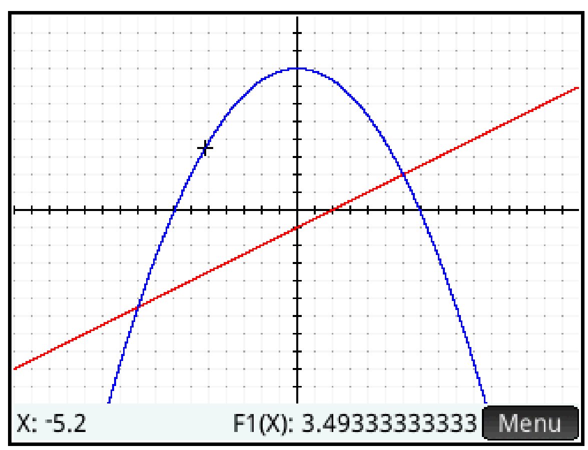


1. Press Apps Info and tap on the Function icon. The app opens in Symbolic view.
2. Enter $8-\dfrac{X^2}{6}$ in F1(X)
3. Enter $\dfrac{X}{2}-1$ in F2(X)
4. For each function, tap on the color picker to choose a color and check/uncheck it to select/deselect it for graphing
5. Press Plot Setup to see the graphs of your checked functions



In Plot view, tap Menu to open the menu. The menu buttons are:

- **Zoom**: opens the Zoom menu
- **Trace**: toggles tracing off and on
- **Go To**: enter a specific *x*-value and the tracer will jump to it
- **Fcn**: opens a menu of analytic functions
- **Defn**: displays the definition of a function
- **Menu**: opens and closes the menu



Things you can do:

- Press  or  to see that you are currently tracing on F1(X)
- Tap anywhere on the display and the tracer will jump to the x-value indicated by your finger tap while still remaining on the function being traced.
- Press  or  to jump from function to function for tracing
- Tap and drag to scroll the graphing window
- Press + Ans and − Base to zoom in and out on the cursor

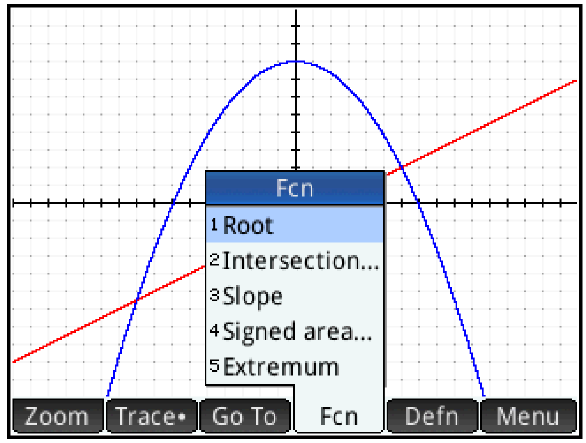
The **Fcn** Menu

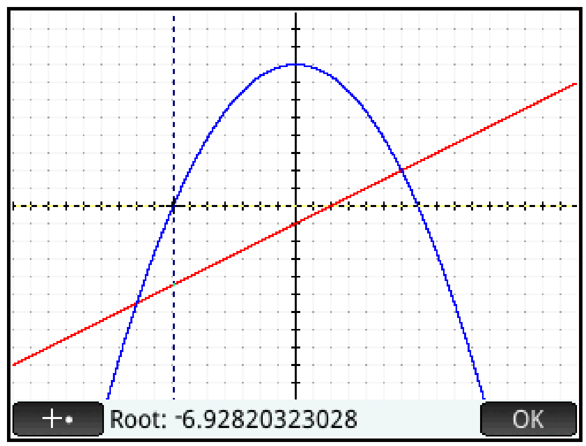
In the following examples, we will use the options in the **Fcn** menu to explore our two functions.

Roots

First, we will find one of the roots of our quadratic function, F1(X).

1. Tap anywhere near the left-most root of the quadratic (around *x*=-7)
2. Tap Menu to open the menu (if necessary)
3. Tap Fcn to open the **Fcn** menu
4. From the list, select Root, either by tapping on it, using the direction keys, or pressing 1 Program Y.
5. The value of the root (*x*=-6.928…) is displayed
6. Press + to show the exact location of the cursor and OK to exit

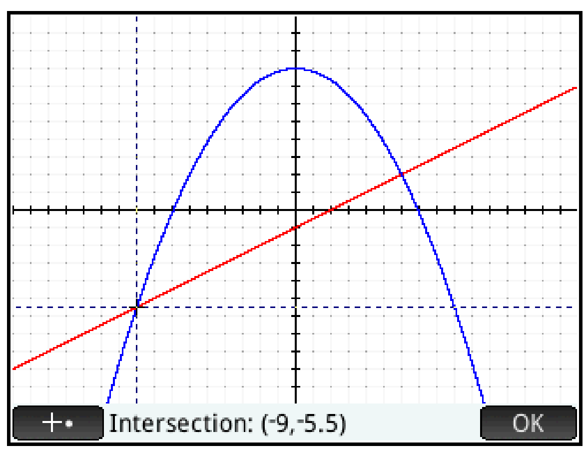




Intersection

We will now find the left-most point of intersection of the two graphs.

1. With the cursor still at the root from the previous example, tap Fcn to open the **Fcn** menu and select Intersection
2. A pop-up menu gives you the choice of finding the intersection with F2(X) or the *x*-axis. Press OK to select F2(X).
3. The intersection is displayed as shown to that right



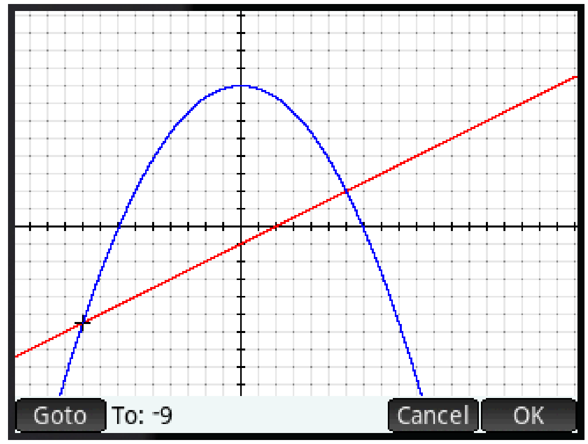
Slope

The Slope option in the **Fcn** menu works similarly to Root, except that the slope continues to be displayed as you trace the function, until Cancel is pressed.

Signed Area

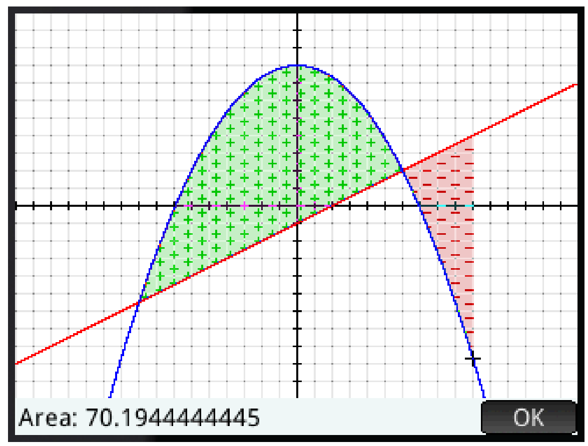
Suppose you wish to find the signed area between the curves, from *x*=-9 to *x*=10.

1. Tap Fcn to open the Functions menu and select Signed Area…
2. Tap near *x*=-9, use the direction keys to move the cursor to *x*=-9 exactly, and tap OK.
3. Now tap near *x*=10 and use the direction keys to move the cursor to *x*=10 exactly



With the touch display, navigation is improved and the experience is more interactive.

As you move the cursor, the area between the curves is filled in graphically. The color display shows you which regions have positive area and which have negative area. The fill patterns have "+" and "-" in them to remind the students that the area is signed.



4. Tap OK to see the area; tap OK again to exit.

Extremum

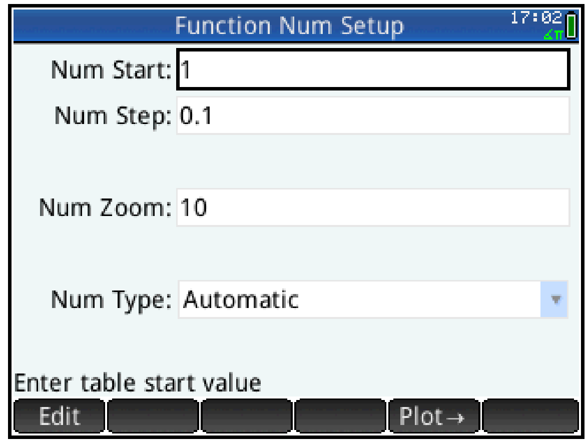
The Extremum option works in a manner similar to the way Root works.

The Function App: App Functions and App Variables

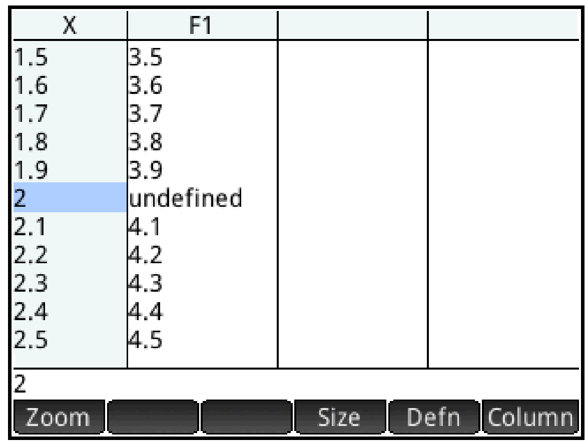
The five functions from the **Fcn** menu are available to you from the Home view and they store their last results in variables named after the functions. For example, in the Home view, ROOT(F1(X),-7) will now return -6.928… and that value will be stored in the app variable Root. The table below lists the most common app functions and app variables for the Function App.

Function App: The App Functions and App Variables			
Fcn Option	App Function Name and Syntax	Example	Stores results in
Root	ROOT(Expr1,Value)	ROOT(X^2-1,0.5)	Root
Intersection	ISECT(Expr1, Expr2, Value)	ISECT(F1(X),3-X,2)	Isect
Slope	Slope(Expr1,Value)	SLOPE(X^2-6,3)	Slope
Signed Area...	AREA(Expr1[,Expr2],Val1, Val2)	AREA(F1(X),-6.9,6.9)	SignedArea
Extremum	EXTREMUM(Expr, Value)	EXTREMUM(F2(X),3)	Extremum

1. Press Symb Setup to return to Symbolic view. We will now look at some of the functionality in the Numeric view of the app.
2. Press Shift Esc Clear to delete all the function definitions. You will be asked to confirm this action. Tap OK .
3. In F1(X), enter $\frac{(X^2-4)}{X-2}$
4. Press Shift Num Setup to see the Numeric Setup view of the app. Change the options as shown in the figure to the right.

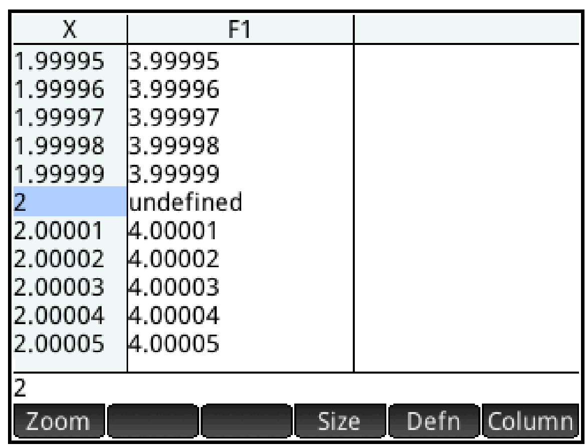


Note the new menu button: PLOT→. If pressed, this button changes the options in this view to match the settings in the Plot view. For example, with the default Plot view, Num Start would be set to -15.9 and Num Step would be set to 0.1. Tracing along the graph in plot view would then mirror navigating through the table: both would show the same (x,y) ordered pairs.



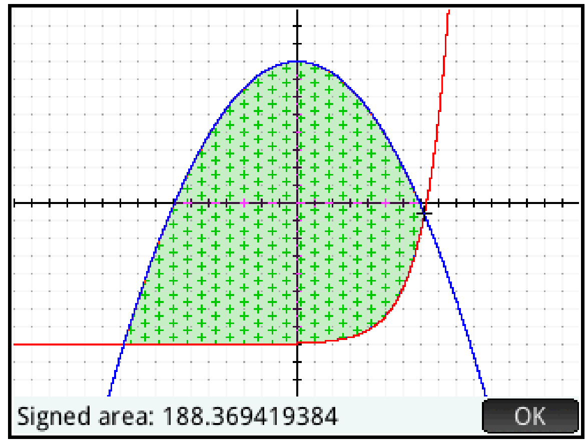
X	F1
1.5	3.5
1.6	3.6
1.7	3.7
1.8	3.8
1.9	3.9
2	undefined
2.1	4.1
2.2	4.2
2.3	4.3
2.4	4.4
2.5	4.5

5. Press Num Setup to see the Numeric view. The menu buttons are:
   - **Zoom**: same as the Plot view menu
   - **Size**: chooses a font size
   - **Defn**: displays the column definition
   - **Column**: chooses 1-4 columns
6. With any value in the x-column selected, type 2 to jump to that value.
7. Now press + Ans and − Base to zoom in and out on that row of the table just as you did to zoom in and out on the cursor in the Plot view.



X	F1
1.99995	3.99995
1.99996	3.99996
1.99997	3.99997
1.99998	3.99998
1.99999	3.99999
2	undefined
2.00001	4.00001
2.00002	4.00002
2.00003	4.00003
2.00004	4.00004
2.00005	4.00005

Example: Dividing Land Equally



Two brothers inherit land that they want to divide equally between them. The local land records indicate that the land is bounded roughly by the two functions $y = 8 - \frac{x^2}{6}$ and $y = \frac{2^{x-1}}{10} - 8$, where x and y are measured in kilometers. If they choose to put up a fence along a border running north and south, where should they put the fence?

1. Enter the functions in F1(X) and F2(X)
2. Use Plot or Numeric view to establish that the two boundaries intersect at x≈ -9.8 and x≈ 7.2
3. Use Signed Area to estimate the area of the land to be 188.37 km^2(18,837 hectares), so each brother should get approximately 94.185 km^2
4. Optional: use Signed Area to get a visual estimation of the boundary
5. Define F3(X) as follows:

$$F3(X) = \int_{-9.8}^{X} F1(T) - F2(T)\delta T$$

6. Use Plot view to get an estimate (x≈ -0.7) and then use Numeric View to zoom in on the solution: x≈ -0.644

X	F1	F2	F3
-1.3	7.718333333	-7.97969369	8.3804396E1
-1.2	7.76	-7.97823624	8.5376238E1
-1.1	7.798333333	-7.97667418	8.6951929E1
-1	7.833333333	-7.975	8.8531125E1
-0.9	7.865	-7.97320566	9.0113480E1
-0.8	7.893333333	-7.97128254	9.1698650E1
-0.7	7.918333333	-7.96922139	9.3286288E1
-0.6	7.94	-7.96701230	9.4876045E1
-0.5	7.958333333	-7.96464466	9.6467574E1
-0.4	7.973333333	-7.96210709	9.8060524E1
-0.3	7.985	-7.95938738	9.9654545E1

-0.7

Zoom | Size | Defn | Column

X	F1	F2	F3
-0.65	7.929583333	-7.96813598	9.4080923E1
-0.649	7.929799833	-7.96811389	9.4096821E1
-0.648	7.930016	-7.96809178	9.4112719E1
-0.647	7.930231833	-7.96806966	9.4128617E1
-0.646	7.930447333	-7.96804752	9.4144516E1
-0.645	7.9306625	-7.96802536	9.4160414E1
-0.644	7.930877333	-7.96800319	9.4176313E1
-0.643	7.931091833	-7.96798100	9.4192212E1
-0.642	7.931306	-7.96795880	9.4208111E1
-0.641	7.931519833	-7.96793658	9.4224011E1
-0.64	7.931733333	-7.96791435	9.4239910E1

94.1763131674

Zoom | Size | Defn | Column

Extensions:

- What if the brothers wanted an east-west boundary? Estimate where that boundary should be.
- The brothers want to divide the land into thirds; where do the north-south boundaries occur?

In this example, we used the Function app to easily visualize and estimate the solution to a typical classroom problem. In our next example (P. 11), we extend that power of visualization.

The Advanced Graphing App

The Advanced Graphing App is designed to plot graphs in the *x*/*y* plane. It can handle conic sections, polynomials, inequalities – virtually any mathematical open sentence in *x* or *y* or both – or neither.

1. Press Apps Info and tap on the Advanced Graphing icon

The app opens in Symbolic view. There are 10 fields (S1-S9 and S0) for defining the graphs you want plotted in the Plot view.

2. In S1, enter $X^2+3Y^2+2XY-81=0$
3. Tap on the color picker to choose a color for the graph
4. Press Plot Setup to see the graph of S1
5. Tap Menu to open the menu

The menu is basically the same as the menu in the Plot view of the Function app, though without the **Fcn** menu.

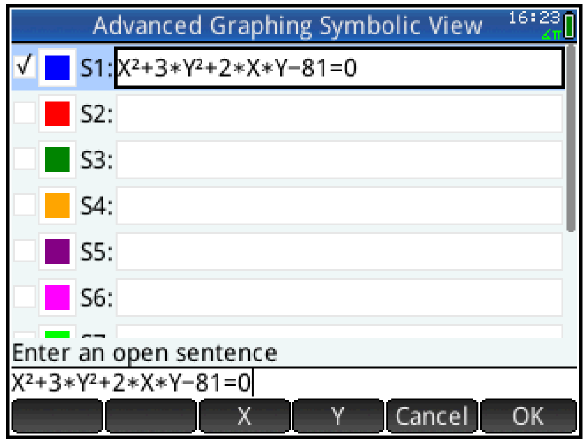
Things you can do:

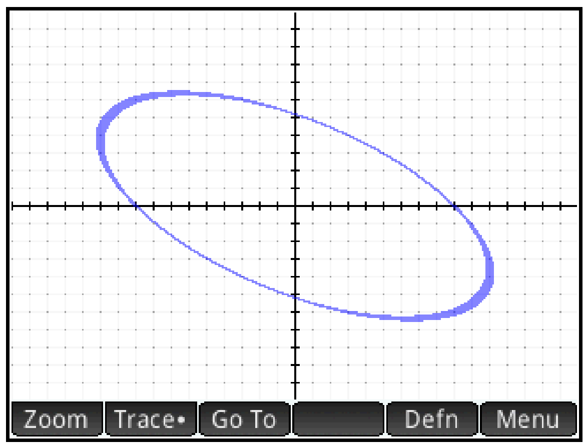
- Tap anywhere on the display to re-locate the cursor
- Press + Ans and − Base to zoom in and out on the cursor
- Tap and drag to scroll the graphing window
- Tap Defn to edit the current relation

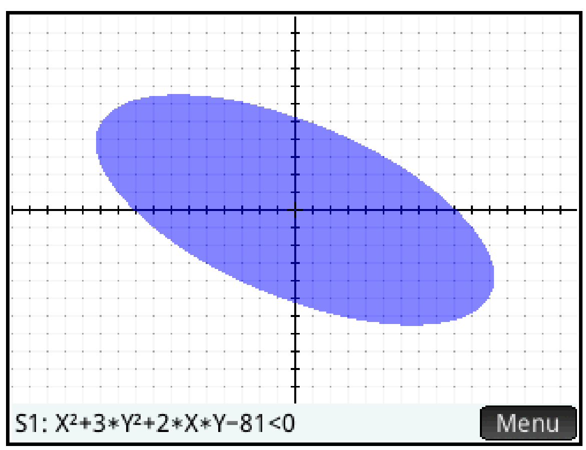
6. Tap Defn and an editor opens, showing you the current expression in textbook format. Tap Edit and change the = to <.

<u>Hint</u>: press Shift 6 ≤,≥,≠ w to open a menu of common relational operators and tap <.

7. Tap OK to see the graph of the inequality and tap ↧ to exit the editor



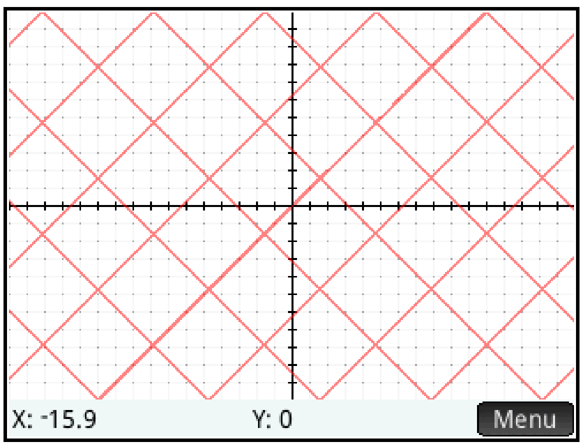




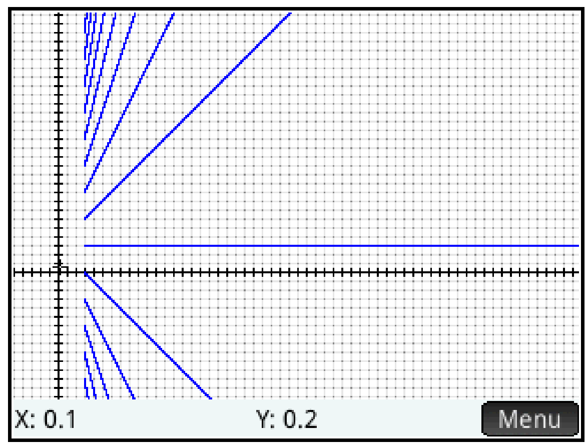
The Advanced Graphing App can plot the graphs of many types of relations. The table below lists just a few.

Relations	Examples	Notes
Polynomials in x and y	$x^2+3x^2+2xy-81=0$	a rotated ellipse
	$4y^4-5x^2y^2+x^4=0$	check out the factors
Linear Inequalities	$2x+3y<5$	
Non-Linear Inequalities	$1>0$	plots every pixel
	$\frac{\lvert x-3\rvert}{2}<4$	
	$Sin\left(\left(\sqrt{x^2+y^2}-5\right)^2\right)>Sin\left(8*Tan^{-1}\left(\frac{y}{x}\right)\right)$	see below

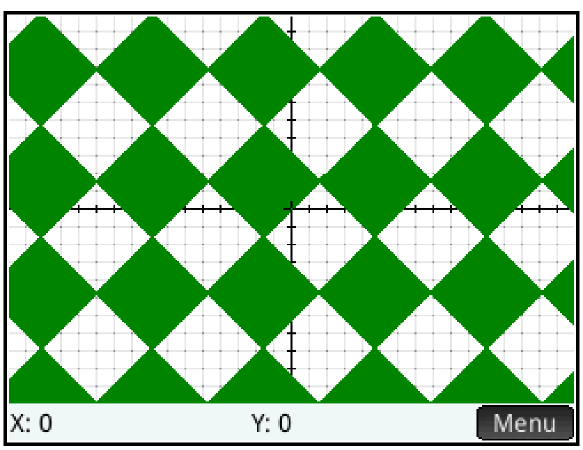
The gallery below shows some example graphs.



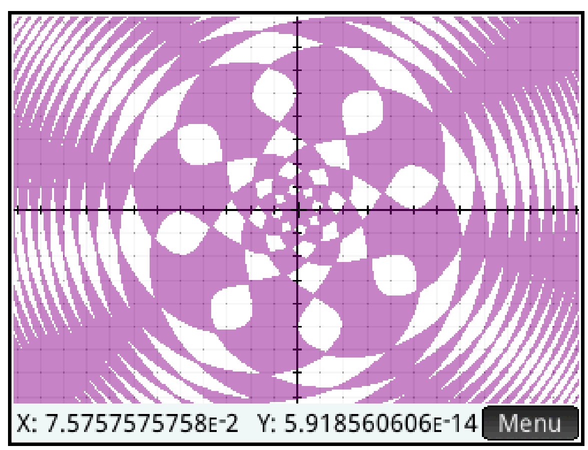
Sin(x)=Sin(y)



y Mod x = 3



Sin(x)<Sin(y)

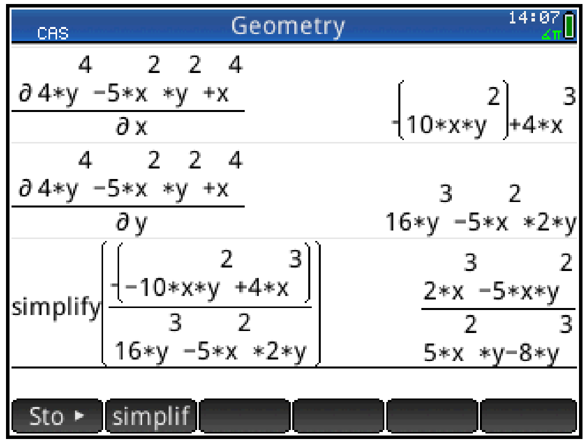


$$Sin\left(\left(\sqrt{x^2+y^2}-5\right)^2\right)>Sin\left(8*Tan^{-1}\left(\frac{y}{x}\right)\right)$$

Example: Implicit Differentiation

If $4y^4 - 5x^2y^2 + x^4 = 0$, find $\dfrac{\delta y}{\delta x}$. This is a typical implicit differentiation problem. The solution shows that the derivative depends on the values of both *x* and *y*, but how do students understand this? In this example, we extend the power of visualization to problems of this sort.

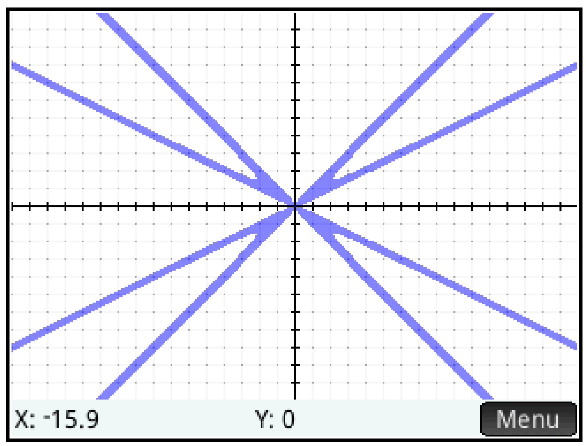
1. Though it will not perform implicit differentiation directly, the CAS does handle it in steps, as shown to the right



Hint: enter the expression first, to keep a copy handy. Use Copy to insert it in your subsequent work.

Now we turn to the Advanced Graphing App to explore further.

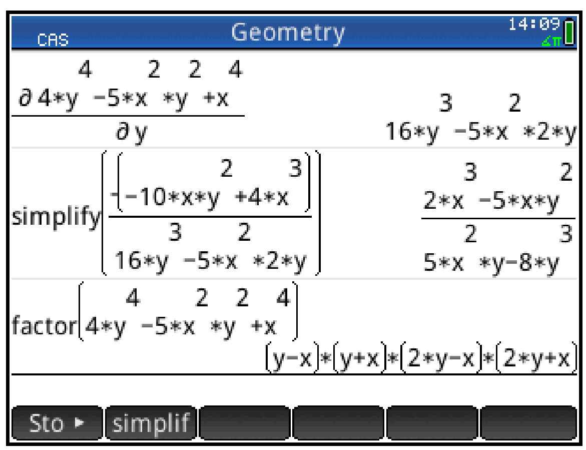
2. Press Apps Info and tap on the Advanced Graphing icon
3. Enter the equation in S1
4. Press Plot Setup to see the graph



The graph appears to be the lines, *y*=*x*, *y*= -*x*, *y*=*x*/2, and *y*= -*x*/2. The CAS factor command gives us a way to verify this.

5. Press CAS Settings to return to the CAS view.
6. Press Mem B ,tap [1]Algebra, and select [4]factor. Then tap on the history and drag back up to your original expression. Tap on it to select it and then tap Copy.

   Press Enter ≈ to see the factors of our expression.

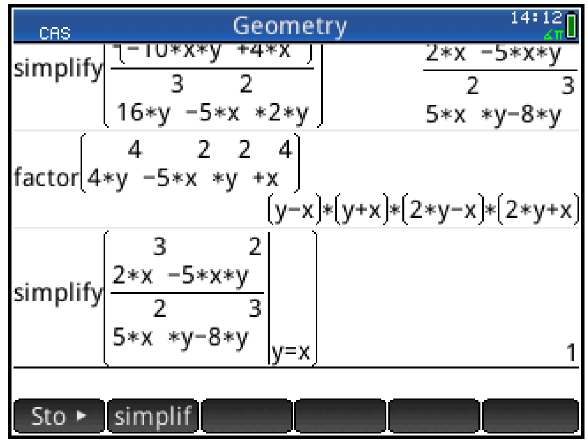


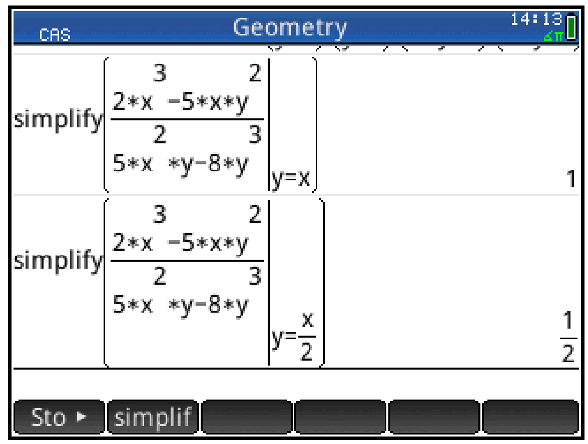
The factors agree with our understanding of the graph. But if the graph consists of those four lines, then the derivative is limited to the values -1, -1/2, 1/2, and 1. How do we reconcile this with the rational expression we have for our derivative?

7. Return to CAS view (CAS Settings) and tap simplif. Press Units, and select □|□ (the Where command). This command is used to make substitutions.
8. Tap and drag the history until you see our derivative expression. Select it and tap Copy .
9. Move to the second box in the template and enter y=x

<u>Hint</u>: use lowercase variables in the CAS

10. Press Enter to see the result.
11. Simply copy the previous input and edit it for the other substitutions (y= x/2, etc.)





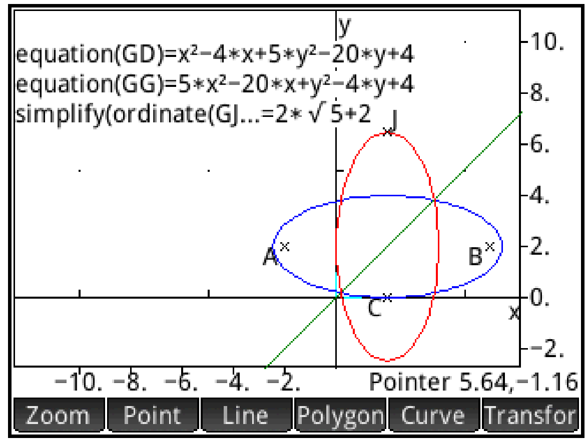
In this example, we extended the power of visualization to include a polynomial in x and y. The graph of the polynomial led to a better understanding of its derivative and to a conjecture regarding its properties, and the power of the CAS let us prove the conjecture. HP Prime, with its CAS and the Advanced Graphing app, represents a new standard in classroom tools to explore mathematics, make conjectures, and pursue proofs.

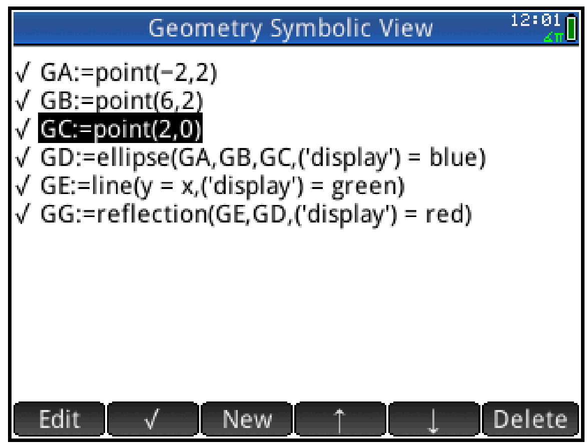
The Geometry App

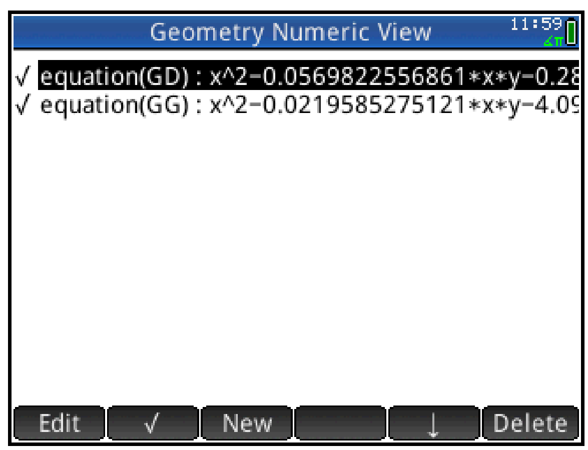
The Geometry app delivers the power of a dynamic geometry app in the palm of your hand. And its CAS integration gives you an analytic geometry app so you can prove your conjectures.

Press Apps/Info and tap on the Geometry icon. The app opens in Plot view. The zoom button (Zoom) opens the familiar Zoom menu with the same options as the other HP apps. Each of the other menu buttons opens menus to create various geometric objects:

- **Zoom**: zoom in or out, etc.
- **Point**: points, points on objects, midpoints, intersections, etc.
- **Line**: segments, lines, tangents, perpendiculars, etc.
- **Polygon**: triangles, quadrilaterals, and special polygons
- **Curve**: circles and other conic sections, locus of points, graphs of functions, etc.
- **Transfor(m)**: translation, reflection, dilation, etc.







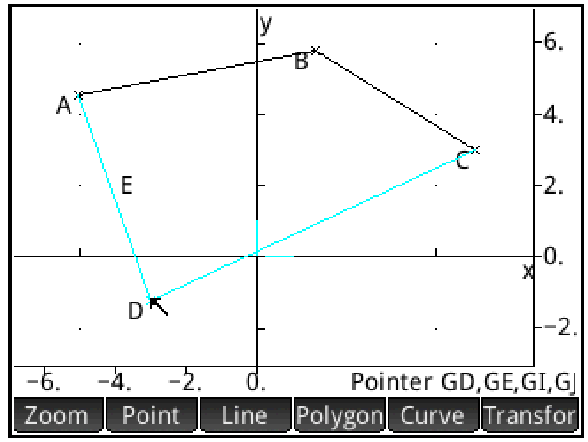
Things you can do:

- Tap and drag to scroll the graphing window
- Press + and − to zoom in and out on the cursor
- Tap any point and press Enter to select it
- Drag any selected point with your finger to move it
- Press Symb/Setup to view, edit, and create symbolic definitions of the objects in Plot view (and to hide or show them)
- Press Num/Setup to view, edit, and create measurements and calculations based on the objects in Symbolic view (and to show or hide them in Plot view)

Example 1: Exploring Quadrilaterals

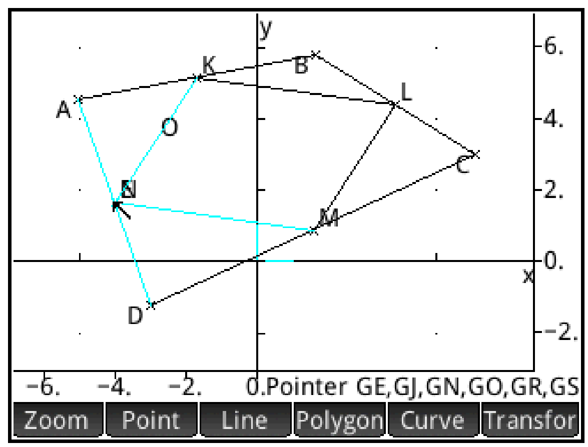
In this activity, we use the Geometry app to create a quadrilateral. We then create and connect the midpoints of consecutive sides of the quadrilateral to form another quadrilateral and explore the properties of the latter in terms of the former.

1. Press Apps Info and tap the Geometry icon. The app opens in its Plot view.
2. Tap Polygon and select ²Quadrilateral.
3. Tap a location and press Enter to select the first vertex of the quadrilateral. Continue to tap and press Enter to select the other three vertices. Press Esc Clear to deselect the quadrilateral command.



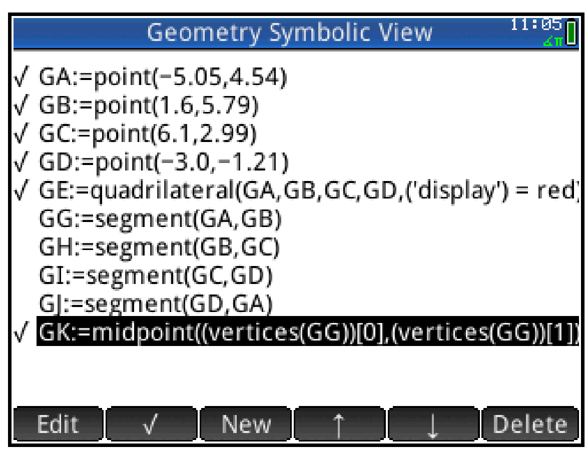
The display now shows a quadrilateral named E, based on the four points A, B, C, and D.

4. Tap Point and select ³Midpoint. Tap near the midpoint of $\overline{AB}$ and press Enter. Continue likewise to create the midpoints of the other segments. When you are done, press Esc Clear to deselect the midpoint command.
5. Repeat Steps 2 and 3 to create quadrilateral O from points K, L, M, and N.

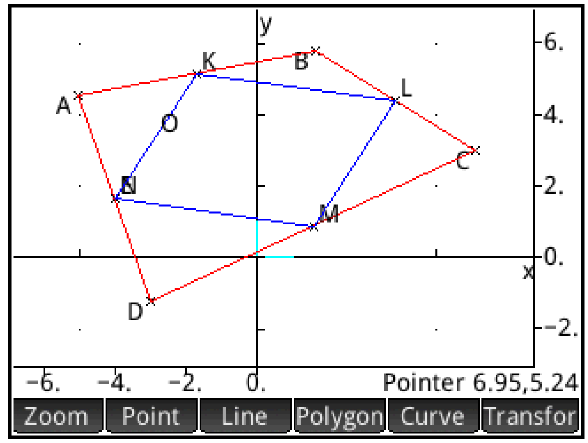


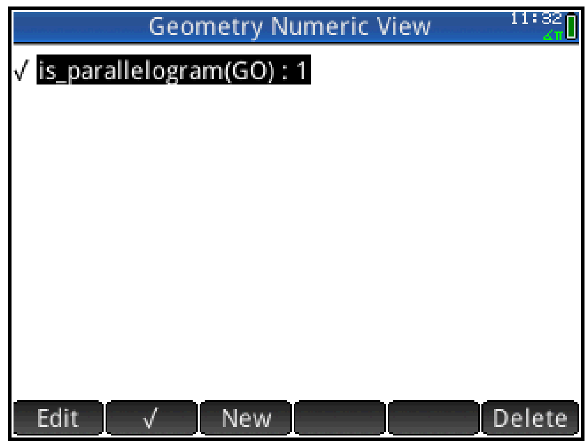
The display now shows both quadrilaterals. We will now tidy up our construction and making it colorful before beginning our explorations.

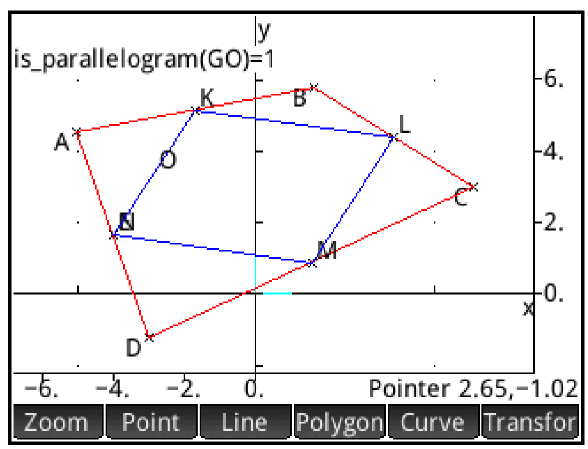
6. Press Symb Setup to display the Symbolic view of the app. Here, each of the geometric objects is defined symbolically. Objects with checks besides them are displayed in the Plot view. Uncheck each segment by highlighting it and pressing √ . Scroll down to uncheck the segments KL, LM, MN, and NK as well.



7. With the segments hidden, return to the Plot view (press Plot Setup) and give the quadrilaterals their own colors. Press Menu Paste to open the context-sensitive menu and select ³Change Color. A choose box pops up with the various geometric objects. Tap on GE:=quadrilateral(GA,...; then tap on the red box in the color picker.
8. Repeat Step 7 to select dark blue for the inner quadrilateral. You will have to drag with your finger to scroll down the choose box of objects to locate GO:=quadrilateral(GK,....
9. Press Num Setup to open the Numeric view of the app. Here we define measurements and tests involving our geometric objects. Tap New to start a new measurement. In this case, it is a test. Tap Cmds to open the menu of commands select ²Tests, then select ᴬis_parallelogram. The command is pasted into the command line. Remember that the name of our inner quadrilateral is GO. Type "GO" between the parentheses and press Enter ≈ . Tap √ to check this test for display in the Plot view.
10. Return to the Plot view to see our constructions and the test result. Drag anywhere outside of the construction to pan the plane so you can see both the test result and the construction.

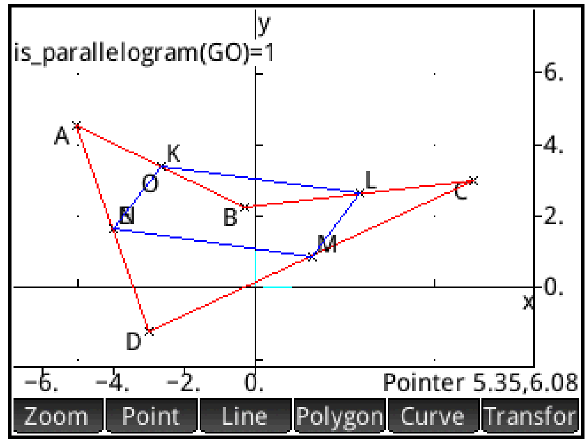






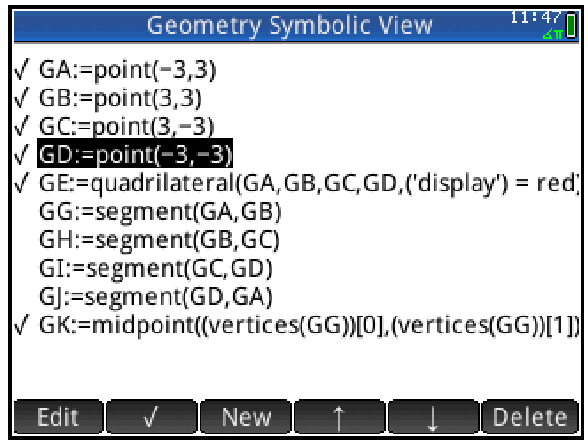
We are now ready to explore our construction.

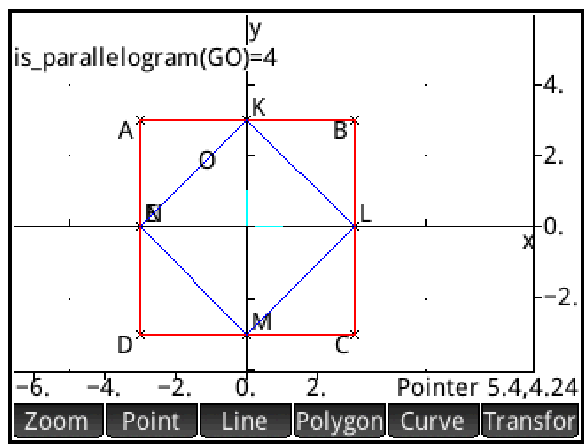
1. Select one of the vertices of the outer quadrilateral by tapping on it and pressing Enter. You can now drag it anywhere within the display with your finger. As you move a vertex, notice that the parallelogram test on KLMN maintains a value of 1, indicating it is always a parallelogram. The parallelogram test can return any of 5 values:
    0. Not a parallelogram
    1. A parallelogram only
    2. A rhombus
    3. A rectangle
    4. A square



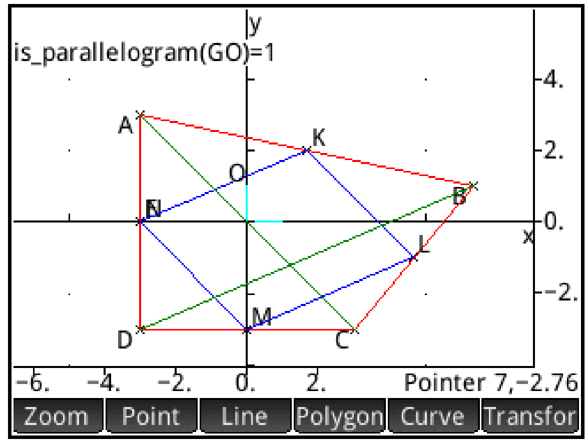
You can now explore by dragging, but there is also another way to explore the effects of the properties of ABCD on KLMN.

2. Press Symb to return to the Symbolic view. Here you can give the coordinates of points A, B, C, and D exact values.
3. Select GA, tap Edit and enter new coordinates (-3,3). Tap OK when you are done.
4. Repeat Step 3 with points B, C, and D so that you have A(-3,3), B(3,3), C(3,-3) and D(-3,-3), making ABCD a square.
5. Press Plot to return to the Plot view. The display shows the parallelogram test now has a value of 4, indicating that KLMN is a square as well.

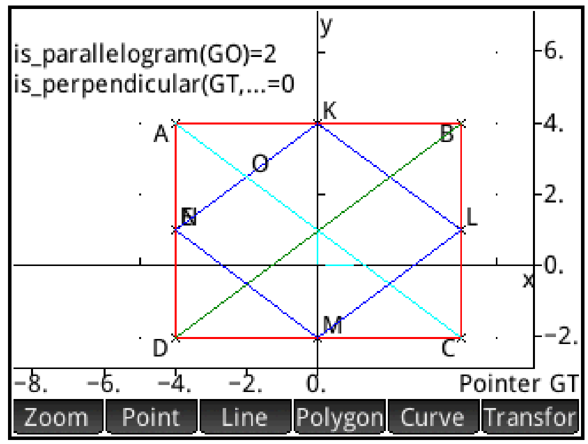




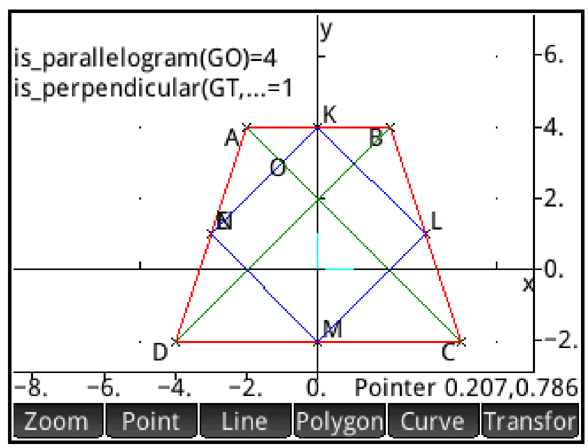
It seems that KLMN is always at least a parallelogram, no matter where we move the coordinates of points A, B, C, and D (as long as they are not collinear!). To see why this is so, simply construct the diagonals AC and BD.



6. Tap Line and select 1Segment. Tap on point A and press Enter. Then tap on Point C and press Enter. Repeat with points B and D. Then press Esc to deselect the command.



Since segment KL joins the midpoints of two sides of ΔABC, then it is parallel to the third side (AC); likewise, MN joins the midpoints of two sides of ΔACD, and is thus parallel to AC as well. Thus KL and MN are parallel. The same can be seen for segments NK and ML.



7. Press Apps and tap Save to save your construction with a name you'll remember

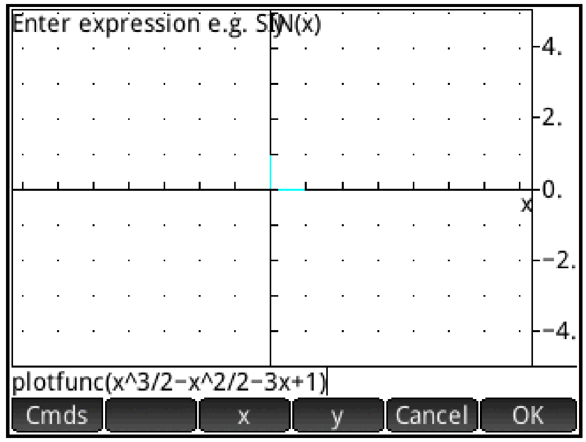
With the testing and symbolic abilities of the Geometry app, students can set about the proofs or refutations of conjectures such as these:

- If ABCD is a rhombus, then KLMN is a rectangle
- If ABCD is a rectangle, then KLMN is a rhombus
- If ABCD is an isosceles trapezoid, then KLMN is a rhombus
- If the diagonals of ABCD are perpendicular, then KLMN is a rectangle
- If the diagonals of ABCD are congruent, then KLMN is a rhombus

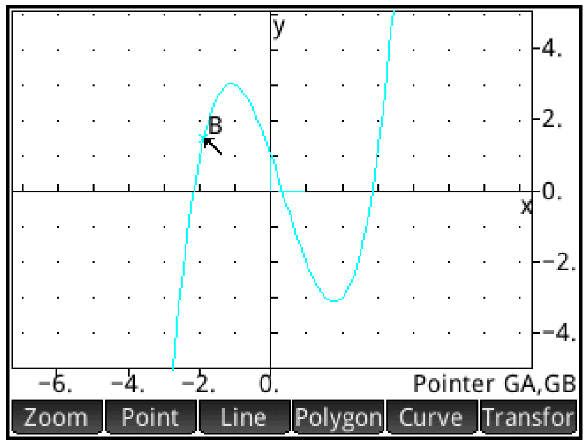
Example II: Slope and Derivative of a Function

In this activity, we construct a visualization tool for the derivative of a function.

1. With your previous construction saved, press Apps Info, use the direction keys to select the Geometry icon, and tap Reset. You will be asked to confirm the reset; tap OK.
2. Tap Curve, tap 6Plot, and select 1Function. If there are any functions defined in the Function app, a window pops up asking if you want to use any of them or create a new one. Select New.

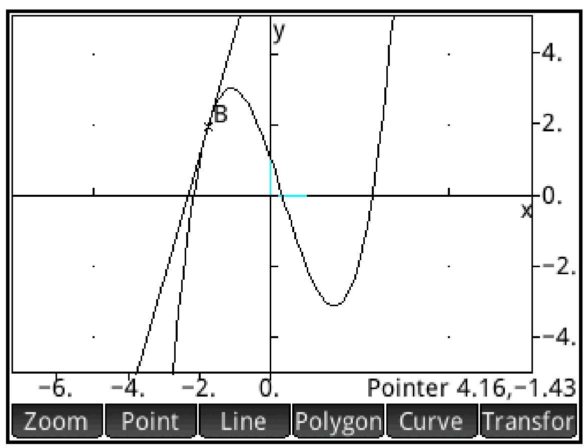


3. An editor opens with plotfunc(and new menu buttons appear with x and y. Enter the function $\frac{x^3}{2}-\frac{x^2}{2}-3x+1$ after the parenthesis and tap OK. The graph of the function is drawn. Press + Ans to zoom in.

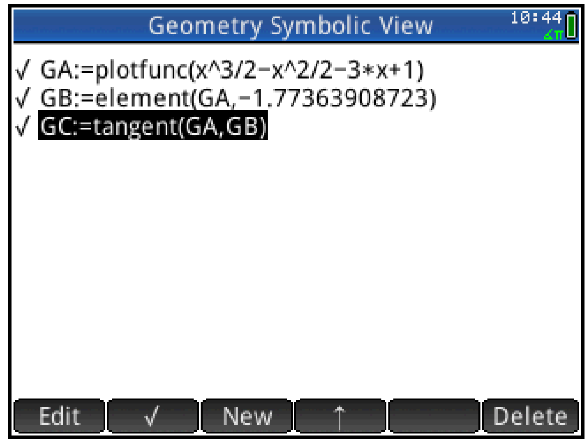


Note: the Geometry app uses lowercase x as the independent variable for functions.

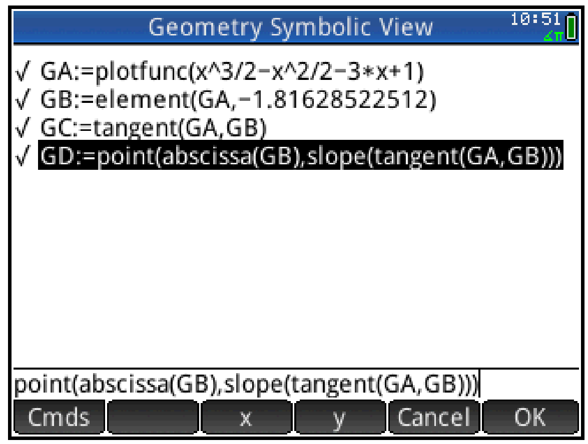
4. Tap Point and select 2Point On. Tap on the graph (it will turn blue when selected) and press Enter ≈. Point B appears, defined as a point on the graph. Press Esc Clear to exit this command.
5. Tap Line, tap 5More , and select 5Tangent. Tap on the curve and press Enter ≈. Then tap on point B. A message at the bottom should read tangent(GA,GB). Press Enter ≈. The tangent to the function through point B is drawn.



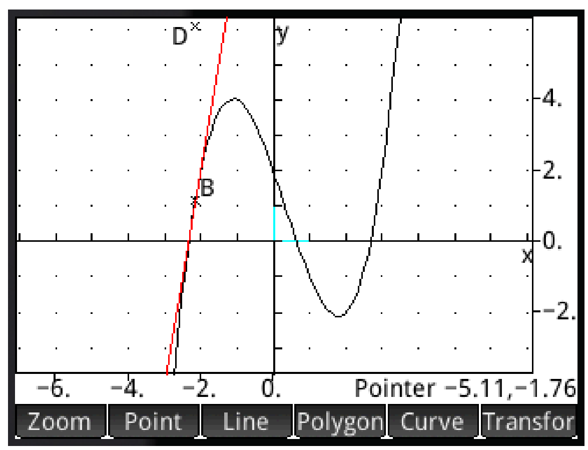
6. Tap point B and press Enter to select it. You can now drag point B along the curve with your finger or tap on a new location and the tangent line will follow.
7. Press Symb to see the Symbolic view of the app. All objects created in the Plot view have their symbolic definitions here.
8. Tap New to create new object. Tap Cmds, tap [1]Point, and select [5]Point. The point() command appears in the edit line.



We will now define the coordinates of this point. The x-coordinate will be the same as that of point B and the y-coordinate will be the same as the slope of the tangent. The new point D will be a point on the derivative of the function.

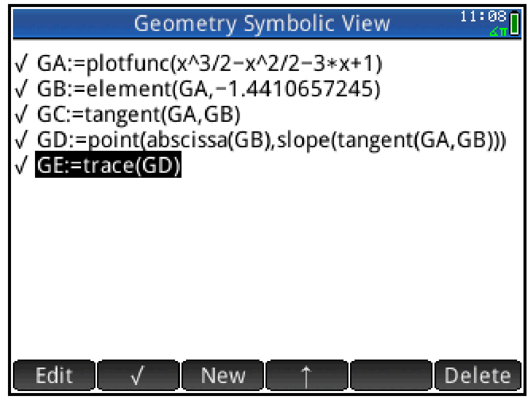


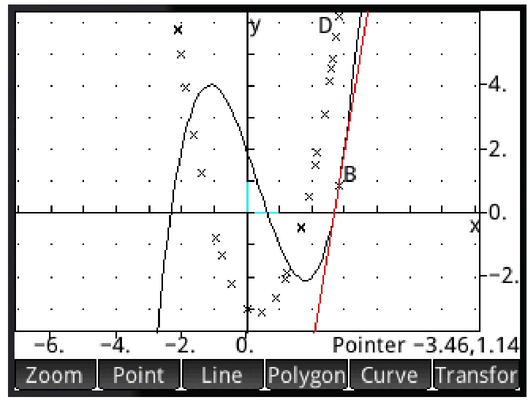
9. Press Mem and tap Catlg to open the Catalog. Press ALPHA Vars to jump to commands that start with A and find abscissa. Tap OK. Between the parentheses, type GB. Move past the right parenthesis, press , and return to the Catalog for the slope and tangent functions. The completed definition is shown to the right.
10. Press Plot to return to Plot view. As you move point B, you can see that point D moves along the derivative.

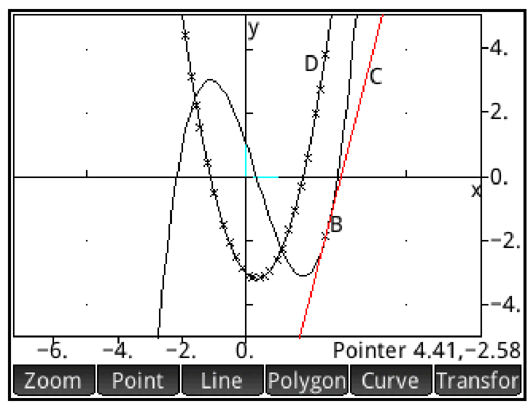


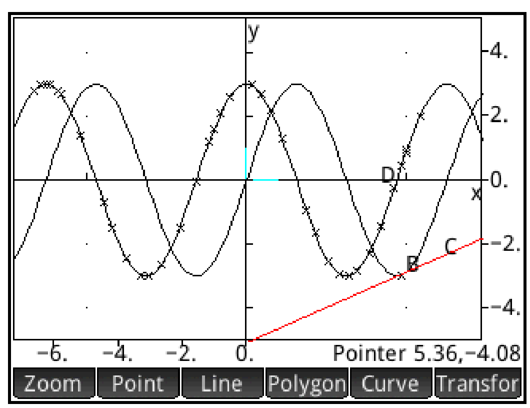
We are now ready to use our construction!

1. Return to Symbolic view and create a new object, GE. As shown to the right, GE:=trace(GD).
2. Return to Plot view, select point B and move it. Each time you tap the display to move point B, a small marker will be left to show where point D was. As you continue to tap, the shape of the derivative function emerges, as shown to the right.
3. Uncheck GE in Symbolic view to erase the trace and keep it from drawing in Plot view
4. Return to Symbolic view and create GF. Define GF:=locus(GD,GB), which means the locus of point D as point B moves along the graph. This new object represents the derivative function, as shown right and below. Check and uncheck it in Symbolic view to show and hide it in Plot view.









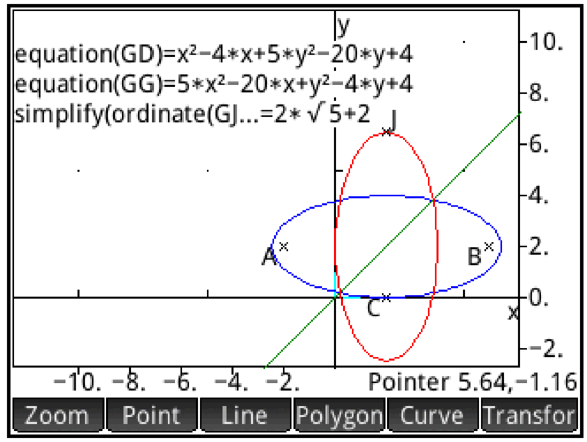
You now have a construction that has a number of methods to visualize the derivative of a function:

- Move point B and see a single point on the derivative that has the same x-value as point B
- Move point B and leave a trace of points on the derivative
- View the entire locus of points on the derivative

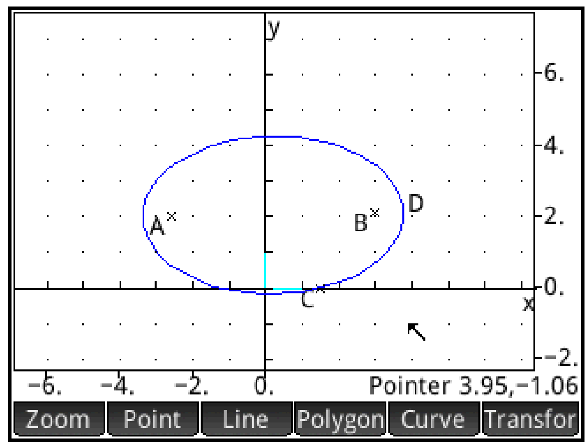
As you can see from the figure to the right, you can return to Symbolic view and change GA to be the graph of any function. The trace and locus will continue to work. Your students can now visualize the graph of the derivative of any function using this one construction. Save your construction now with a name you will remember!

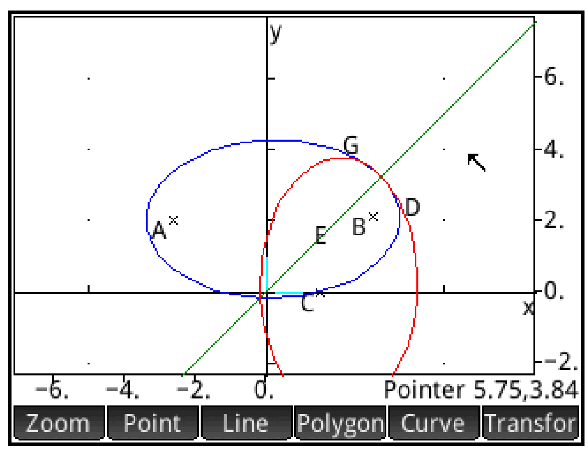
Example III: Reflection and Inverse

In this construction, we have a blue ellipse defined by the focal points A and B, and a point C on the ellipse. The red ellipse is the reflection of the blue one over the line y=x, so they are inverses of each other. Note the equations of the two ellipses and how the roles of *x* and *y* are interchanged. Since the points A, B, and C were defined exactly in the Symbolic view, the equations are exact as well. Also, the y-coordinate of point J is shown to emphasize the exact nature of the result.



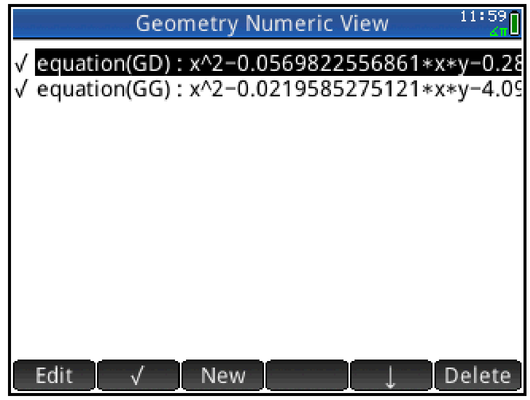
1. With your previous construction saved, reset the Geometry app
2. Tap Curve and select ²Ellipse. Tap at the location of the first focus and press Enter. Then proceed with the second focus and a point on the ellipse. Press + to zoom in and drag to scroll the window. Change the color of your ellipse to blue.
3. In Symbolic view, create GE:=line(y=x)
4. Back in Plot view, tap Transfor and select ²Reflection. Tap the line and press Enter. Then tap the ellipse and press Enter. Each time you select an object, it should turn light blue to indicate it is selected. Press Esc to exit the reflection command. Change the color of the reflected ellipse to red.





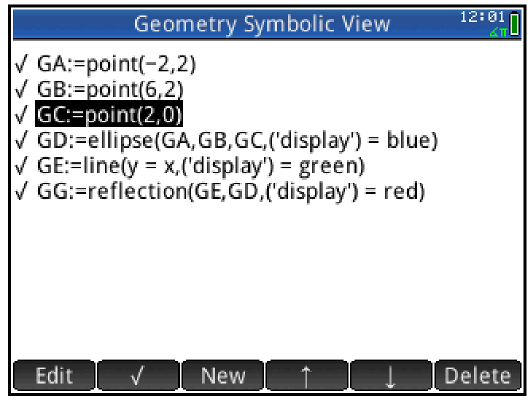
The Numeric view of the Geometry app is where you go to make measurements or create calculations based on measurements. Check the ones you want displayed in the Plot view.

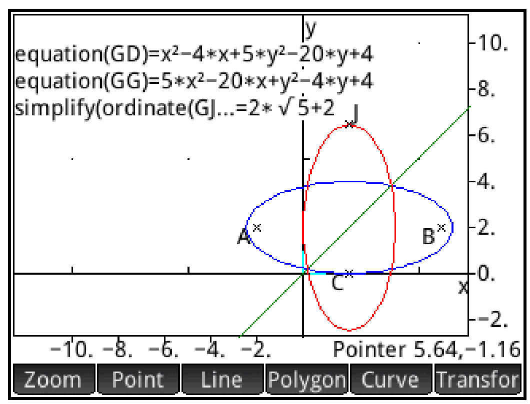
5. Press Num Setup to see Numeric view. Tap New to create a new measurement or calculation. Tap Cmds to open the Commands menu, tap 1Measure, and select Bequation. After the parentheses, type GD (the name of the first ellipse) and tap OK.
6. Repeat Step 5 to get the equation of the reflected ellipse (GG)
7. Check both entries in Numeric view to ensure that they are visible in Plot view.



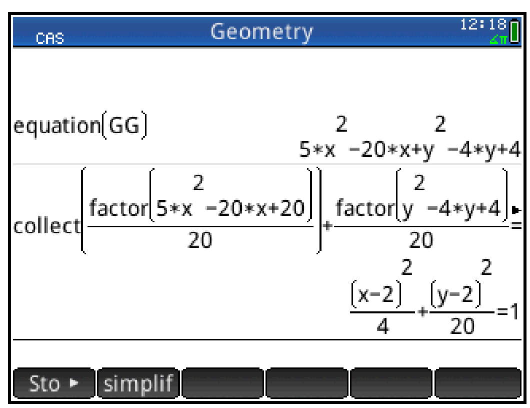
Note the approximate nature of the equations.

8. Return to Symbolic view and change the coordinates of points A, B, and C as shown to the right above.
9. Now when you return to Plot view, the equations are shown exactly.





You can see that the roles of x and y are interchanged in the two equations. Note also that the y-coordinate of Point J is reported as an exact value as well. The CAS can be used to retrieve the equation of the red ellipse and put it into a more familiar form. Note the length of the major semi-axis is $\sqrt{20} = 2\sqrt{5}$ and the center is at (2,2). So the point J on the reflected ellipse should have a y-coordinate of $2 + 2\sqrt{5}$, which it does! The integration of CAS and dynamic geometry gives you an exact analytical geometry app that is dynamic as well.

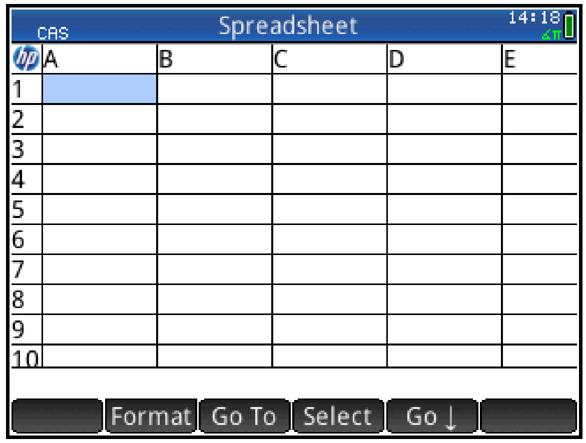


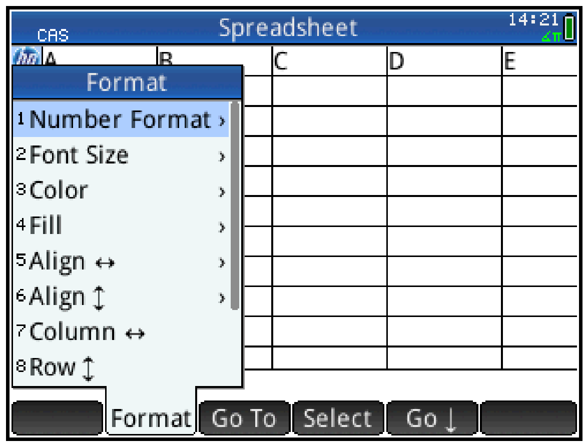
The Spreadsheet App

The Spreadsheet App gives you the most common features you expect in a spreadsheet. But with HP Prime, you also get the power of a CAS integrated with the spreadsheet.

Press Apps Info and tap the Spreadsheet icon. The app opens in Numeric view-its only view. The menu keys are:

- **Format**: opens the Format menu (see figure to the right and below)
- **Go To**: jumps to a specified cell
- **Select**: activates Selection mode
- **Go**: determines which cell is selected when you tap OK
- **Show**: displays the cell contents in textbook format





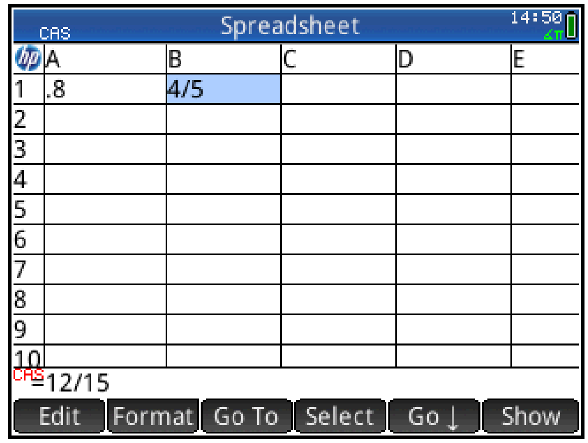
Things you can do:

- Tap and drag to scroll through the spreadsheet
- Tap and hold to invoke Selection mode; then drag to select a rectangular block of cells
- Pinch open or closed horizontally to change the width of a column
- Pinch open or closed vertically to change the height of a row
- Enter contents into a cell (number, matrix, expression
- Define a cell, row, column or sheet using a formula

The Spreadsheet app can return numerical approximations for a formula, or it can use the CAS to return exact numeric or symbolic results. The examples in this section will cover both uses.

To get started in the Spreadsheet app, here is a simple exercise that illustrates the CAS and non-CAS uses of the app.

1. Select cell A1, type 12/15, and press Enter.
2. Select cell A2 and type the same thing, but before pressing Enter, tap CAS. The dot next to CAS shows the CAS is now active

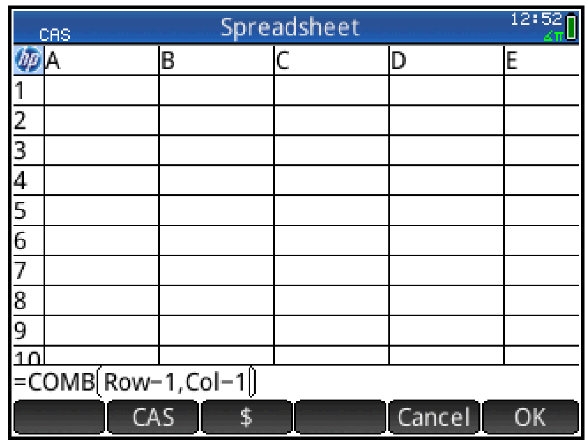


A1 is evaluated numerically, but A2 is simplified using the CAS. Select B1 to see the acronym CAS at the start of its formula.

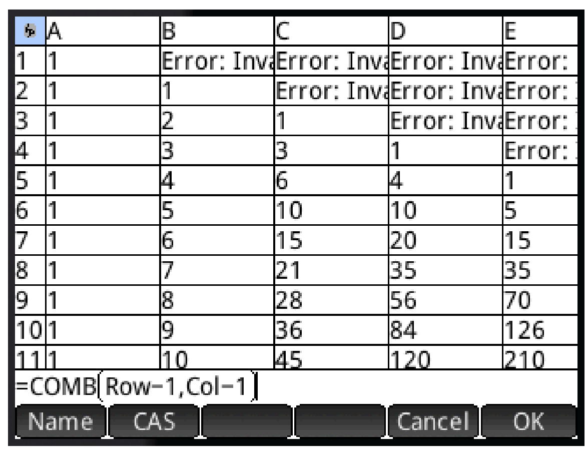
Example 1: Pascal's Triangle

Next we define the entire spreadsheet with a single command.

1. Tap the upper-left corner to select the entire sheet. Press Shift = to start a new formula. Then press Mem B, tap Math, tap 5Probability, and select 3Combinations. Between the parentheses, enter Row-1,Col-1, as shown to the right.



<u>Hint</u>: Row and Col are Spreadsheet app variables. To retrieve Row, press Vars, tap App, tap 1Spreadsheet, tap 1Numeric, and select 3Row. Select Col in a similar manner. You can always just type names in letter by letter, using ALPHA for uppercase and ALPHA Shift for lowercase letters.



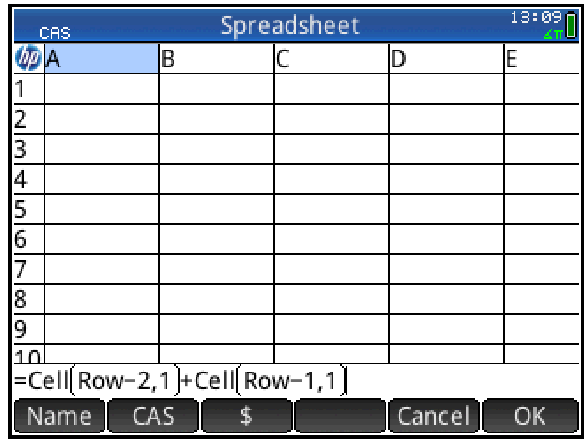
2. Tap OK to see the spreadsheet fill with Pascal's triangle! Use your finger to scroll through the spreadsheet.
3. To clear the entire spreadsheet, tap on the upper-left corner and press Shift Esc.

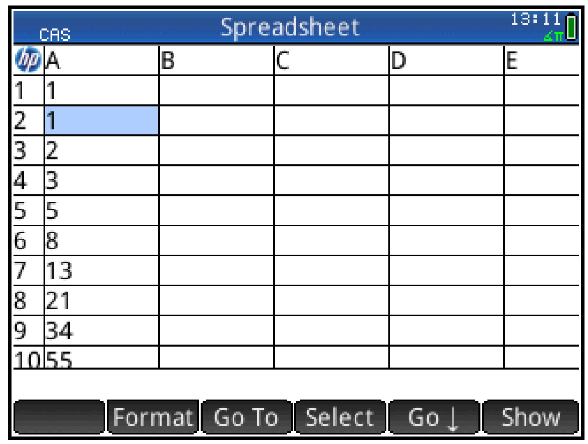
Example II: The Fibonacci Sequence

Another important variable for the Spreadsheet app is Cell, as you will see in our next example.

1. Tap on the column header for Column A to select it.
2. Press Shift = to start a new formula. Then enter Cell(Row-2,1)+Cell(Row-1,1) as shown to the right.
3. Tap OK to see Column A fill with zeroes.
4. But now enter 1 in cell A1 to see Column A fill with the Fibonacci sequence.

You can now appreciate the value of the app variables, Cell, Col, and Row!

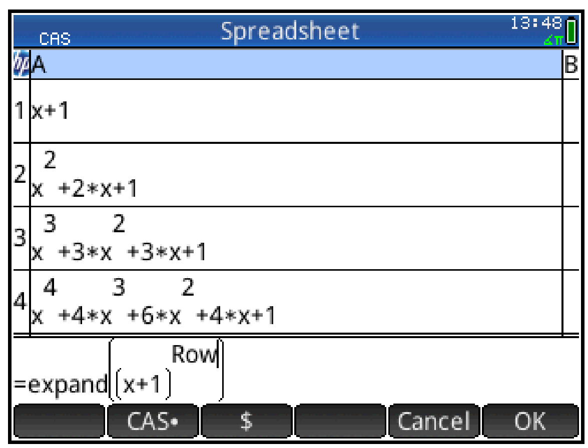




Example III: Back to Pascal's Triangle

Here is another CAS example that involves symbolic results.

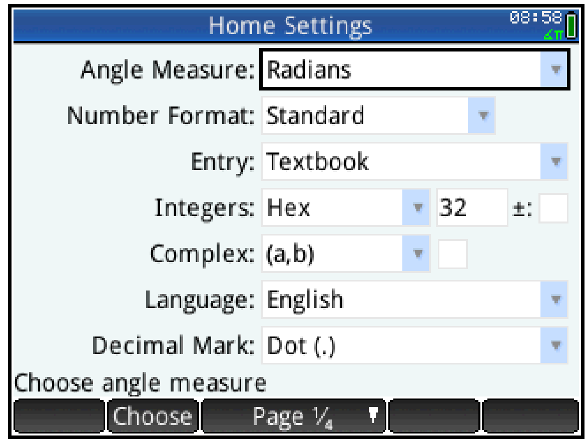
In the example to the right, we use the CAS to define one column of the spreadsheet to expand the binomial x+1 to various integer powers. Note the expression editor shows a CAS button: CAS. When active, the CAS is used to evaluate the formula. When it is not active, the formula is used to obtain numerical results.

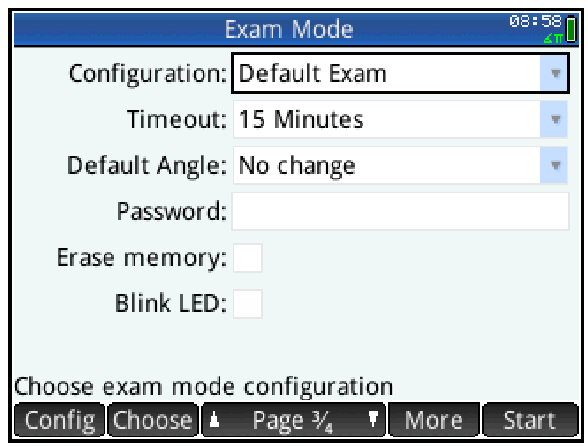


Imagine all the patterns you and your students can explore in symbolic expressions!

Examination Mode

HP Prime can be configured and locked for an examination. The machine will remain locked for a pre-set time period and secured with a password. LED lights at the top of the unit will flash to show that it is in examination mode.





1. Press Shift Settings to enter Home Settings
2. Swipe upwards to get to the third page; the header will say *Exam Mode*. Alternately, press the right halves of Page 1/3 and Page 2/3.

The menu keys are:

- **Config**: opens the Configuration page, where you can check which features you want disabled
- **Choose**: opens a choose box
- **Page**: tap the left help to go up a page and the right half to go down a page in the Home Settings
- **More**: opens a menu of options to copy or reset the current configuration
- **Start/Send**: starts Exam mode on the current HP Prime or send it to another HP Prime (**Start** changes to **Send** if Prime is connected via USB)

Things you can do:

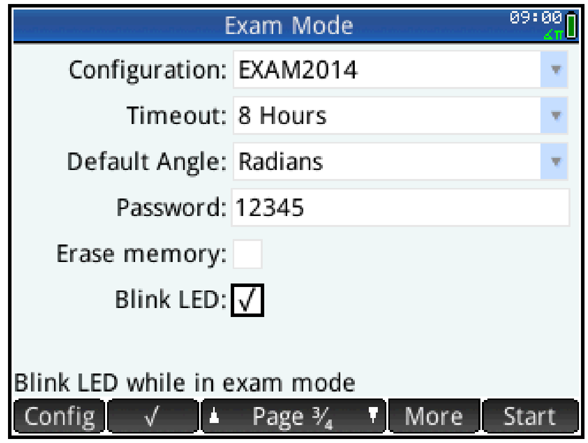
- Give your configuration a name
- Set a time period
- Set a password
- Check a box to erase memory when examination mode starts
- Check a box to make the LED lights blink while in examination mode.

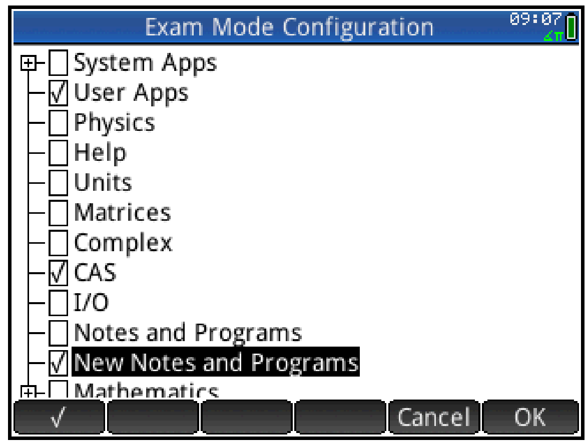
3. Tap *Configuration*, tap More , and enter a name for our new configuration: EXAM2014
4. Tap *Timeout*, then tap Choose and select a time period.
5. Tap *Default Angle*, then tap Choose and select a default angle measure
6. Tap *Blink LED* twice (to select it and check it)

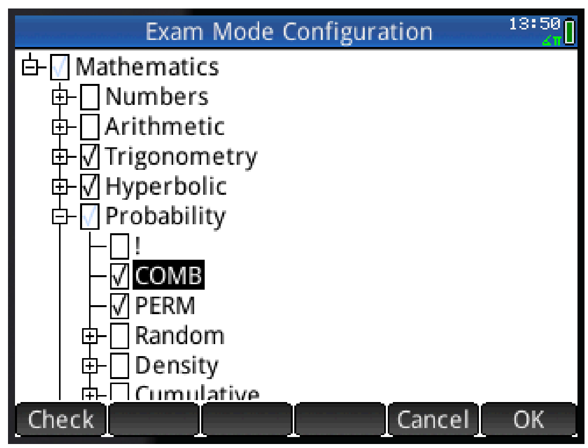
We are now ready to define our EXAM2014 configuration.

7. Tap Config to enter the Configuration page
8. Tap *User Apps* to disable saved apps with their data
9. Tap *CAS* to disable the CAS
10. Tap *New Notes and Programs* to disable the Note and Program editors
11. Tap and drag to scroll down the tree
12. Tap on the plus sign (+) next to *Mathematics* to expand the tree
13. Tap on *Hyperbolic* to disable all hyperbolic trigonometric functions
14. Tap on the plus sign next to *Probability* to expand the tree another level and tap *COMB* and *PERM* to disable the individual functions $_nC_r$ and $_nP_r$
15. Tap OK to save this configuration with your new name: EXAM2014
16. Press Start to start Exam Mode on the device, or if it is connected to another HP Prime, press Send to start Exam Mode on the attached HP Prime

Once Exam Mode starts, the LED lights will blink to show that the configuration is in effect. All HP Prime calculators that were sent the same Exam Mode configuration from the same HP Prime will blink in unison using a random pattern of the 3 color lights.





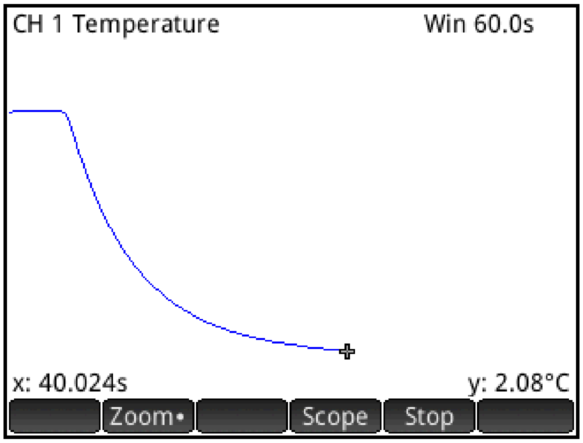


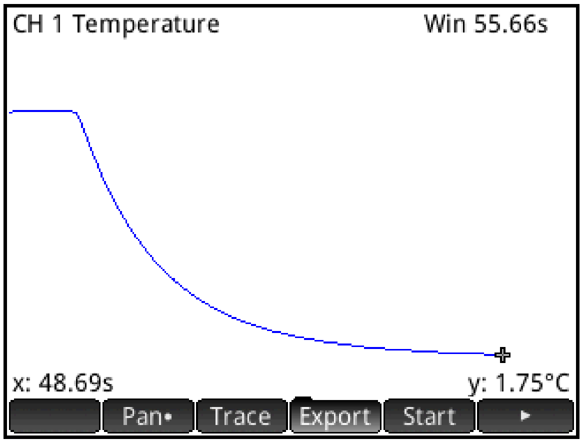
The Data Streamer App

The Data Streamer App works with the HP StreamSmart 410 and up to four Fourier® sensors to collect data in real time. The final data set is sent to one of the statistics apps for analysis. Just plug 1-4 sensors into the StreamSmart 410 and start the Data Streamer App.

Press Apps Info and tap the DataStreamer icon. The app opens in Plot view. The menu keys are:

- **Chan**: select one of the 4 channels for tracing (if more than one sensor is connected)
- **Pan/Zoom**: select whether the direction keys are used to Pan (scroll) or Zoom the Plot view
- **Trace**: toggles tracing off and on
- **Scope**: start oscilloscope mode
- **Export**: opens a menu to select data to export to a statistics app for analysis
- **Start/Stop**: starts and stops data streaming
- ▶: displays a second page of options





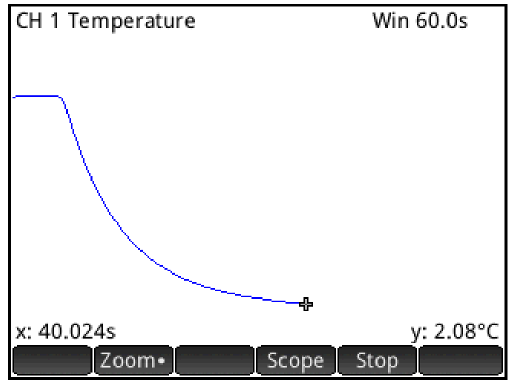
Things you can do:

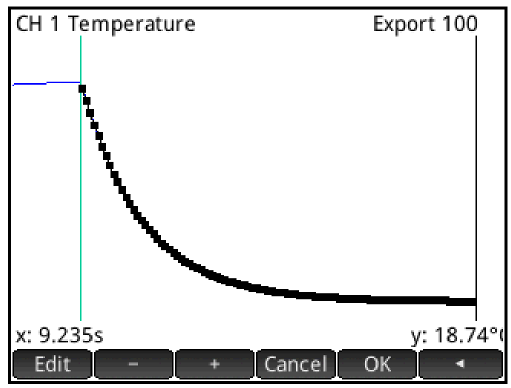
- With Pan active, press ▲ and ▼ to move the current data stream up and down in the Plot view
- Tap Pan• to change it to Zoom . Press ▶ and ◀ to zoom in or out on the Plot view while data is streaming. The effect is to speed up or slow down the data stream until it meets your needs.
- Tap Stop to stop data streaming. It changes to Start , ready for you to start another data stream
- Tap Export to open the Export menu, where you can select just the data you want and send it to one of the Statistics apps for analysis

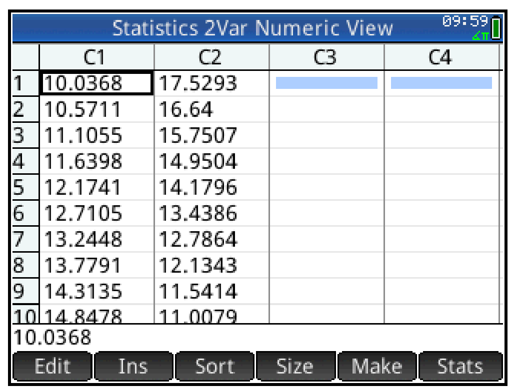
Example: Temperature Experiment

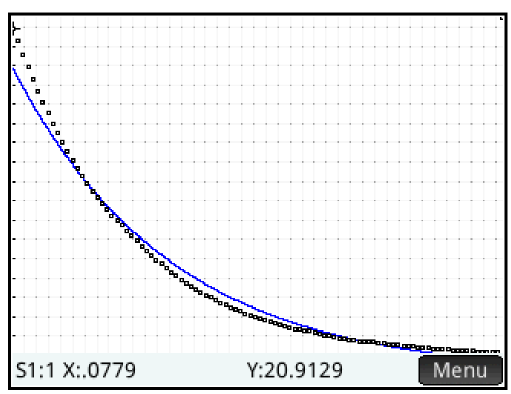
In this example, we plunge a temperature sensor into a beaker of ice water.

1. Attach a temperature sensor to the StreamSmart 410, and attach the StreamSmart 410 to your HP Prime via the USB cable
2. On the HP Prime, Press Apps Info and tap the DataStreamer icon
3. Tap Start to begin data streaming
4. While streaming and with Pan active (Pan•), press ▲ and ▼ to center the data stream in the display
5. Tap Pan• to change it to Zoom . Now press ▲ and ▼ to zoom in and out vertically so the stream fits in the display
6. When you see the data you want, tap to stop data streaming
7. Tap Export to open the Export menu.
8. Tap [and press ▶ and ◀ to crop data from the left
9. When you have selected the data you want, tap ▶ and OK . Tap OK again to start the Statistics 2Var app with your data.
10. Press Symb Setup to enter the Symbolic view. Tap on *Type1*, tap Choose and select Exponential.
11. Press View Copy and select Autoscale to see the scatter plot of your data and the exponential fit.
12. Press Symb Setup to return to the Symbolic view to see the equation of your fit



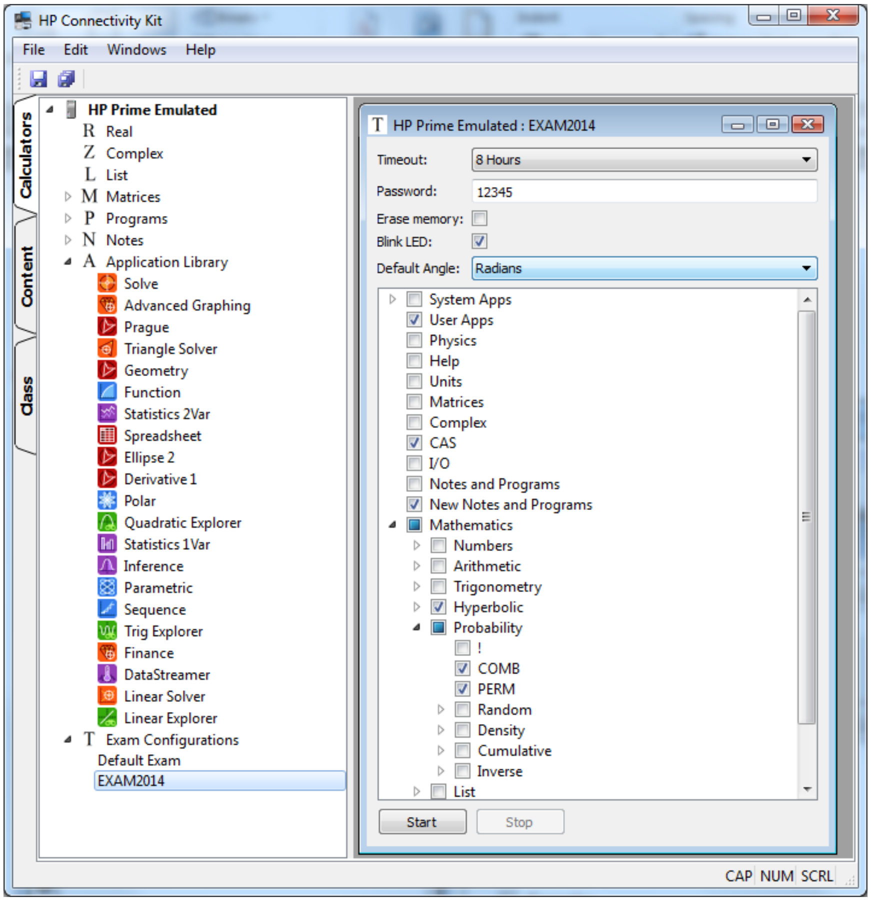






The HP Prime Connectivity Kit

The HP Connectivity Kit allows you to connect to one or more HP Prime calculators via USB or wirelessly.



The Connectivity Kit has three panes:

- **Calculator Pane**: see the data on the connected calculator, edit apps, write programs, and synchronize the new data with the connected HP Prime calculator
- **Content Pane**: create and edit Exam Mode configurations, create polls and quizzes, etc.
- **Class pane**: see all HP Prime displays in your classroom network, monitor students, send apps, data, polls, quizzes, share one student's display for discussion purposes, send and start an Exam Mode configuration to the entire class, etc.

Appendix

Example: Local Linearity, the Difference Quotient and Limits

This activity uses the Function App in a split-screen view to explore the basic concept of local linearity and the limit of the difference quotient as underpinning the notion of derivative.

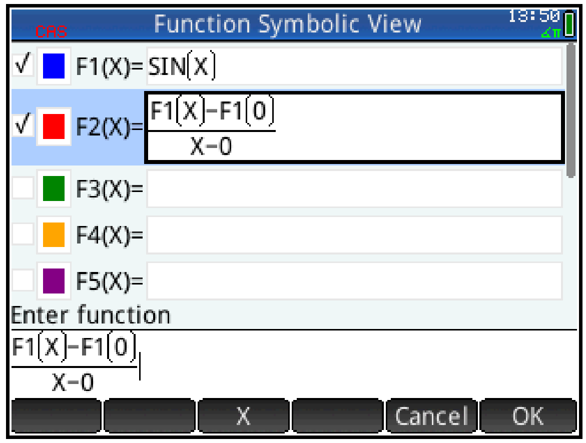
11. Press Apps Info and tap the Function icon.

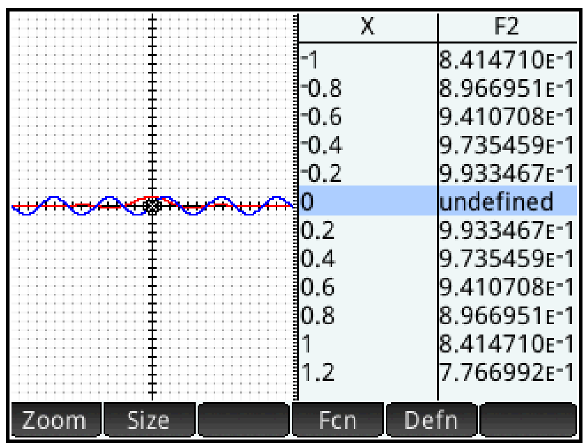
    The app opens in its Symbolic view.
12. Enter $F1(X) = \sin(X)$ and

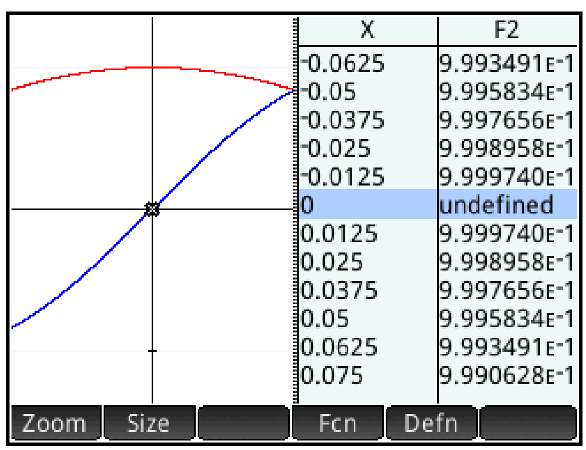
    $F2(X) = \dfrac{F1(X) - F1(0)}{X - 0}$ as shown to the right. We use “X-0” in the denominator to reflect the difference quotient and to allow extensions.
13. Press View Copy and select Split Screen: Plot Table. Press 0 Enter to create a row in the table at x=0, then press ⊙ to trace F2(X). The table shows the value of F2(0) as undefined. The cursor appears to remain on F1(0), since the value of F2(0) is not defined.
14. Press + Ans to zoom in on the graph of F1(X) at x=0.

As you continue to press + Ans to zoom in, you are zooming in on both the graph of F1(X) at x=0 and the row of the F2(X)-values where x=0. Connecting both of these simultaneously helps students see that:

- F1(X) begins to exhibit local linearity near x=0
- The slope of F1(X) near x=0 appears to be converging to 1 as a limit

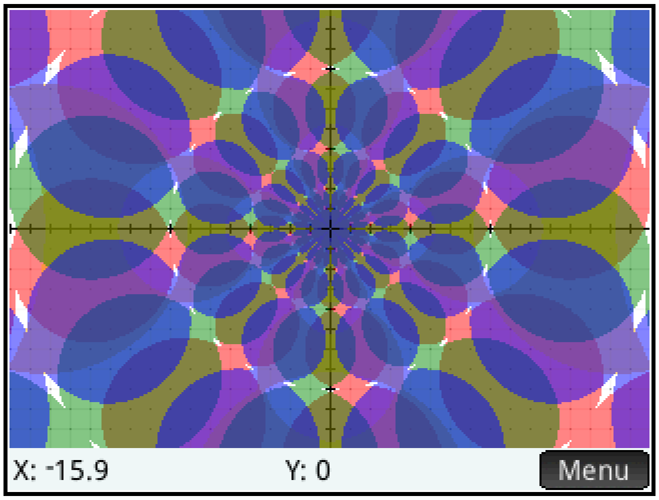






Example: Color blending with multiple relations plotted in the Advanced Graphing app

The graphic below was created using 4 relations plotted in the Advanced Graphing app. It shows how color blending is used with multiple relations in the app. Change the color of each relation and observe the effect on the graph. The relations are shown below. If you are using the HP Prime virtual calculator, you can copy and paste them easily.



S1:
(SIN((π/LN(2))*LN(√(X^2+Y^2)*(3*COS(((ATAN(X/Y)+(π/12))MOD(π/4))-(π/8))+√(9*COS(((ATAN(X/Y)+(π/12))MOD(π/4))-(π/8))^2-8))))*SIN((π/LN(2))*LN(√(X^2+Y^2)*(3*COS(((ATAN(X/Y)+(π/12))MOD(π/4))-(π/8))-√(9*COS(((ATAN(X/Y)+(π/12))MOD(π/4))-(π/8))^2-8)))))<0
S2:
(SIN((π/LN(2))*LN(√(X^2+Y^2)*(3*COS(((ATAN(X/Y)+(-π/12))MOD(π/4))-(π/8))+√(9*COS(((ATAN(X/Y)+(-π/12))MOD(π/4))-(π/8))^2-8))))*SIN((π/LN(2))*LN(√(X^2+Y^2)*(3*COS(((ATAN(X/Y)+(-π/12))MOD(π/4))-(π/8))-√(9*COS(((ATAN(X/Y)+(-π/12))MOD(π/4))-(π/8))^2-8)))))<0
S3:
(SIN((π/LN(2))*LN(√(X^2+Y^2)*(3*COS(((ATAN(X/Y)+(0/12))MOD(π/4))-(π/8))+√(9*COS(((ATAN(X/Y)+(0/12))MOD(π/4))-(π/8))^2-8))))*SIN((π/LN(2))*LN(√(X^2+Y^2)*(3*COS(((ATAN(X/Y)+(0/12))MOD(π/4))-(π/8))-√(9*COS(((ATAN(X/Y)+(0/12))MOD(π/4))-(π/8))^2-8)))))<0
S4:
(SIN((((π/LN(2))))*LN(2*√(X^2+Y^2)*(6*COS(ATAN((((MIN(ABS(X),ABS(Y))/MAX(ABS(X),ABS(Y)))))))+√(36*COS(ATAN((((MIN(ABS(X),ABS(Y))/MAX(ABS(X),ABS(Y)))))))^2-27))))*SIN((((π/LN(2))))*LN(2*√(X^2+Y^2)*(6*COS(ATAN((((MIN(ABS(X),ABS(Y))/MAX(ABS(X),ABS(Y))))))))-√(36*COS(ATAN((((MIN(ABS(X),ABS(Y))/MAX(ABS(X),ABS(Y)))))))^2-27)))))<0

HP PRIME GRAPHING CALCULATOR

CONTENT

INDIVIDUAL ACTIVITIES

· Mathematical Exploration with HP Prime
· ……
· …..

Mathematical Exploration with HP Prime

Sometimes True, Always True, Never True

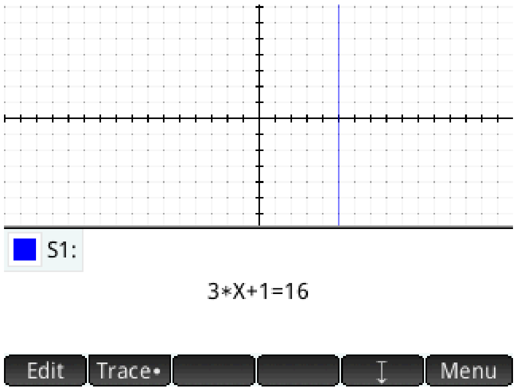
If we look at the statement $3x + 1 = 16$ we can say immediately that this is true only when $x = 5$. Compare that to the statement $2x + 3x = 5x$. That is true for any value of x. It doesn't matter what x is, 2 times a number plus 3 times the same number is always the same as five times that number. Now compare with the statement $\sin(x) = 2$. There is no value of x where $\sin(x) = 2$ because the values of $\sin(x)$ are always between negative and positive one.

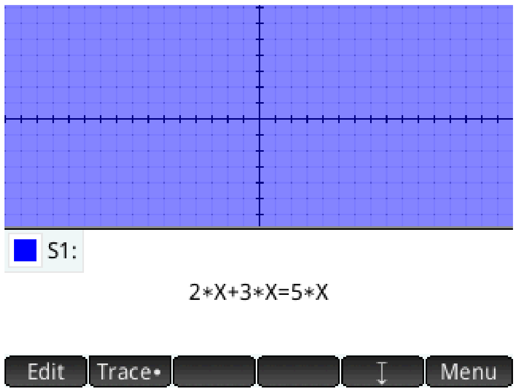
So:

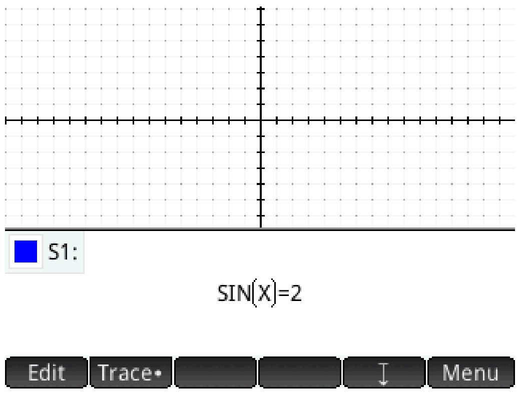
- $3x + 1 = 16$ is sometimes true … in fact true in one instance $x = 5$
- $2x + 3x = 5x$ is always true … it is true for all x
- $\sin(x) = 2$ is never true.

See what happens when we type these statements into the Advanced Graphing App in HP Prime.

1. Click Apps
2. Select Advanced Graphing
3. Type the statement into the line for S1
4. Click Plot
5. Press Menu then Defn (you can drag the graph into a more suitable position if you like)
6. Click Symb to edit the statement, press backspace to delete, then type the next statement and continue as before.







The graph for $3x + 1 = 16$ is a graph of $x = 5$. These two statements are the same. The solution is $x = 5$

The graph of $2x + 3x = 5x$ is everything. It is always true.

The graph of $\sin(x) = 2$ is nothing. It is never true.

Mathematically we are very interested in situations of the first two types. An equation which is always true is called an **identity**. Essentially the statements are identical. Equations which are sometimes true have **solutions** and we are interested to know which equations have solutions and how many they might have.
In both cases we can take the statement and manipulate the algebra to see if we can find statements which are solutions or are identical.
For $3x + 1 = 16$ the solution is $x = 5$
But $2x + 3x = 5x$ is always true so it is an identity. We write it: $2x + 3x \equiv 5x$

Note: Just because the screen shows all, some or none shaded does not prove that these are the only outcomes, just in the range that the screen is showing. You could zoom out (press the – key a few times, which is more convincing, but you cannot see an infinite range!) Nonetheless this gives a very good visual indication.

Mathematical Exploration with HP Prime

Activity 1

Test these statements with the Advanced Graphing App. Are they sometimes, always or never true? In each case, if it is sometimes true, what is/are the solution(s)? If it is always true, then the statement is an identity, so re-write it with the identity symbol. If it is never true, explain why.

1. $2\sin(x) = 2$ (remember the calculator is probably set to radians)
2. $x^2 = x + 6$
3. $x^2 = x - 6$
4. $\frac{1}{x+2} = 0$
5. $\frac{1}{x+2} = 1$
6. $5x(x-3) + 2(x+1) = 5x^2 - 13x + 2$
7. $3xy - 2yx = xy$
8. $x^2 - y^2 = (x+y)(x-y)$
9. $x^2 + 6 = y - 5x$
10. $(x-1)^2 = 4 - y^2$

Note: In written algebra when we write xy we mean $x \times y$. However, xy is a perfectly good name for a single variable and so there is a possible confusion. In computer systems you should always type $x \times y$ i.e. X*Y

We hope that you noticed that with one variable (just x), 'sometimes true' gave one or more solutions. However, with both variables involved (x and y) 'sometimes true' gave a relationship. One was a circle, one was a parabola. With one statement on its own we have three types of outcome:

1. An equation with (or without) solutions;
2. An identity;
3. A relationship (most generally called mapping).

Activity 2

Use the advanced graphing App to construct your own examples in each of these three categories.

Equation		Identity	Mapping
With solution(s)	Without solution(s)		

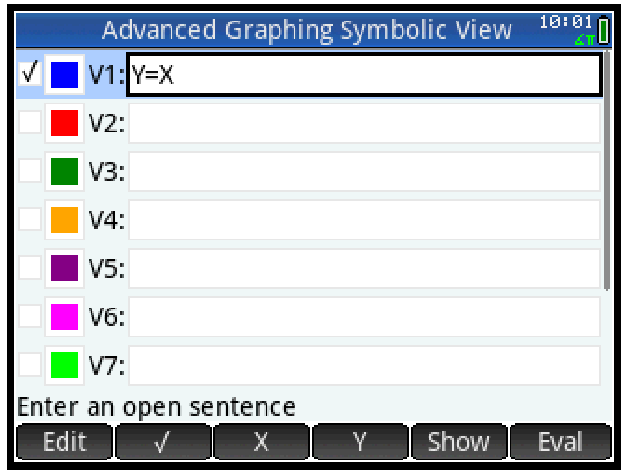
Extend the table as you explore.

System of Linear and Quadratic Equations with HP Prime

Remember a system of linear equations can have exactly one solution, many solutions, or no solution. There are also a variety of solutions to a system of linear and quadratic equations. You will discover and investigate these solutions.

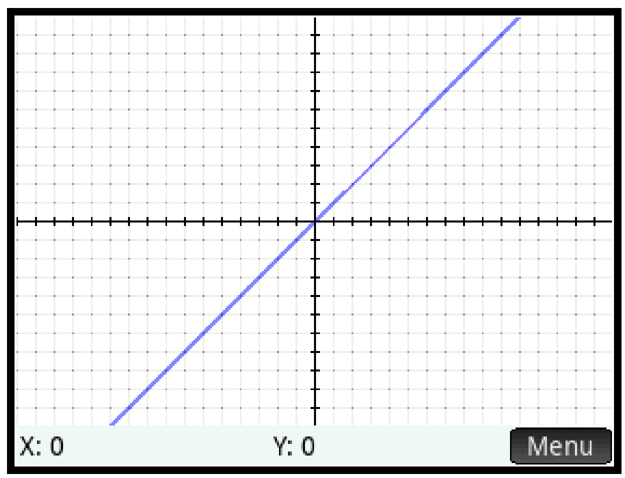
For the Student

1. Press Apps Info to open the App Library. Choose the Advanced Graphing App. The app opens in symbolic view. Clear all equations by pressing Shift Esc Clear if needed.



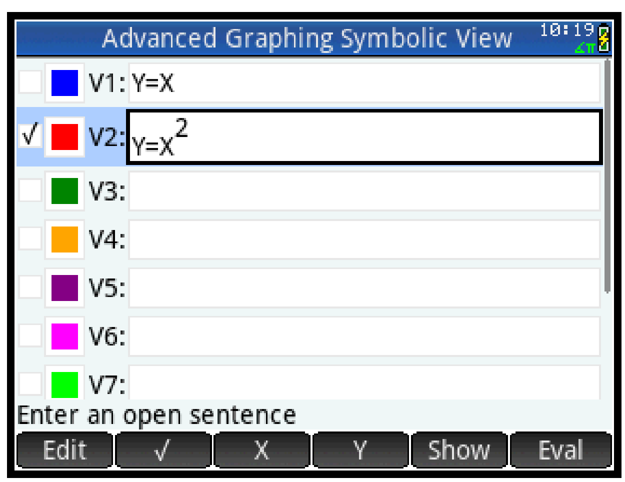
2. First you will look at a linear equation. In V1, enter the linear equation shown on the screenshot. Tap the menu keys Y = X. Tap OK. Your equation should have a checkmark next to V1. If this is missing tap √.

3. Press Plot Setup to view the graph. Find the slope, y-intercept, and x-intercept.

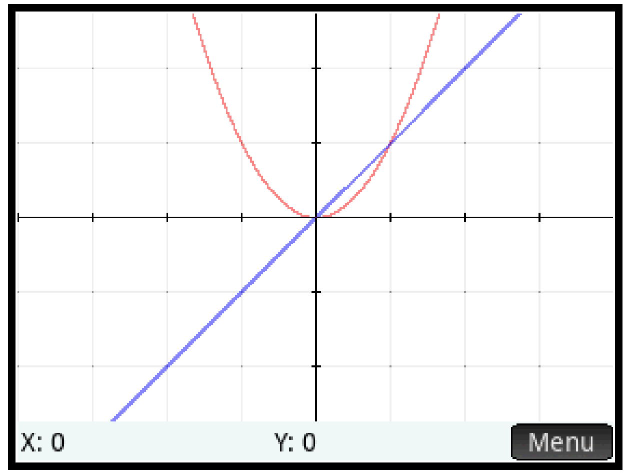


   Slope:
   y-intercept:
   x-intercept:

4. Press Symb Setup. Now enter the quadratic equation shown on the screenshot. Deselect the check by V1. V2 should be selected. This will hide the linear equation while you look at the quadratic equation.



5. Press Plot Setup to view the graph. Find the vertex, roots (x-intercepts), and y-intercept.
   Vertex:
   Roots:
   Y-intercept:



6. Press Symb. Select V1. Press Plot to view the system in plot view. Press + to zoom in on the graph. Notice that the linear and quadratic graphs intersect at two points. These points are called solutions to the system. Find the points.

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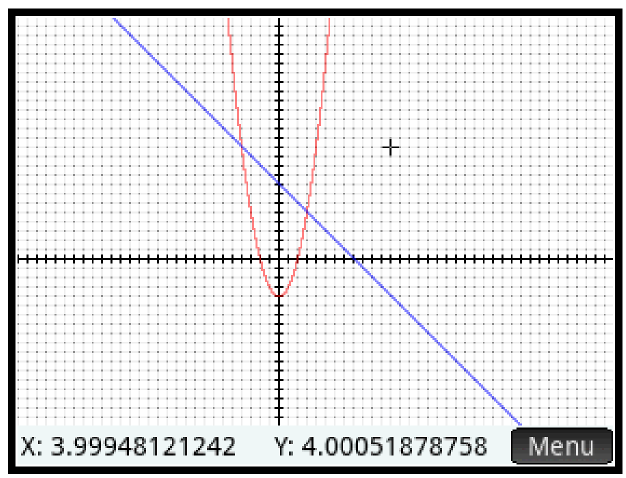
7. Prove using algebra that the two points you found above are solutions to both the linear and quadratic equations.

$y = x$ $\qquad\qquad$ $y = x^2$



8. Remember a system of two linear equations has a variety of solutions so does a system of linear and quadratic systems. Next you will be graphing these systems and determining the number of solutions (intersections). Use V1 for the linear equation and V2 for the quadratic equation.

9. In V1 enter $y = -x + 8$. In V2 enter $y = x^2 - 4$. Press Plot to view the system. Press − to zoom out. Determine the number of solutions.

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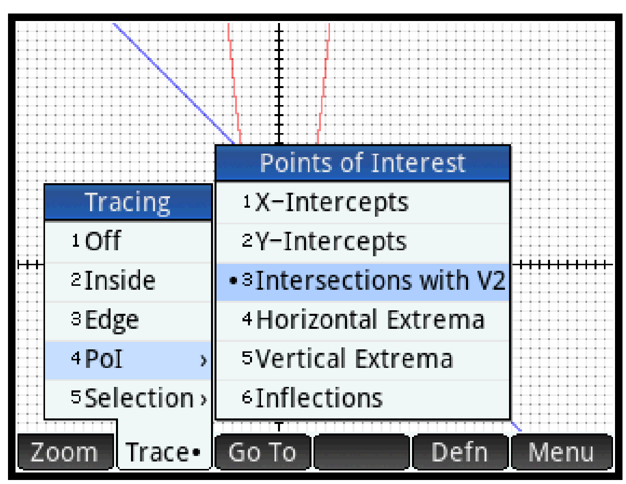

10. To find the coordinate points where these solutions occur tap Menu. Now tap Trace• select POI then Intersections with V2. The solution ordered pair will be shown on the bottom of the screen. Press ▲ ▼ to toggle between solutions. What are the solutions to the system?

____

11. Return to the symbolic view by pressing Symb Setup. Clear all equations.

12. In V1 enter $y = -2x - 1$ and V2 enter $y = x^2 - 4x$. Press Plot Setup. Determine the number of solutions.

____

What are the solutions?

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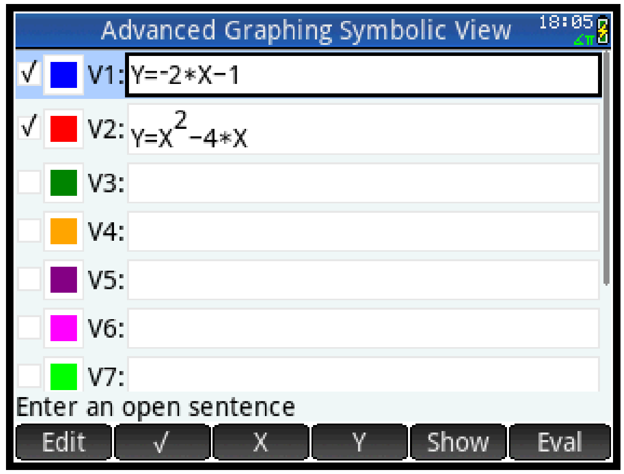


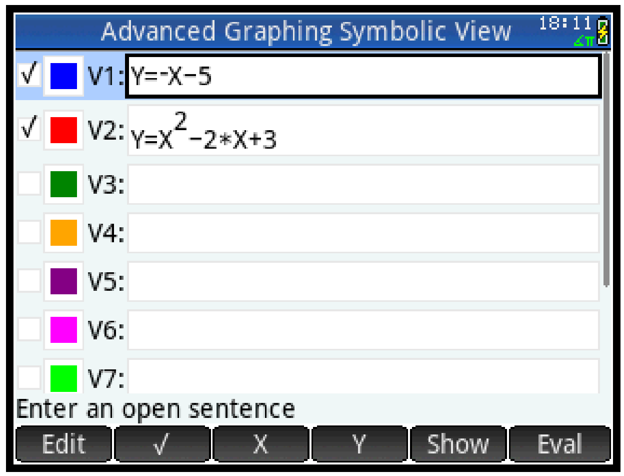
13. Return to the symbolic view by pressing Symb Setup. Clear all equations.

14. In V1 enter $y = -x - 5$ and V2 enter $y = x^2 - 2x + 3$. Press Plot Setup. Determine the number of solutions.

____

What are the solutions?

____



15. Now determine if the same is true for a horizontal line and quadratic equation. Choose any horizontal equation $(y = \_\_)$ for V1. In V2 enter $y = x^2 - 6$. Translate your horizontal line using different values to determine the types of solutions. Sketch each type of solution on the graph provided.

What types of solutions did you discover?

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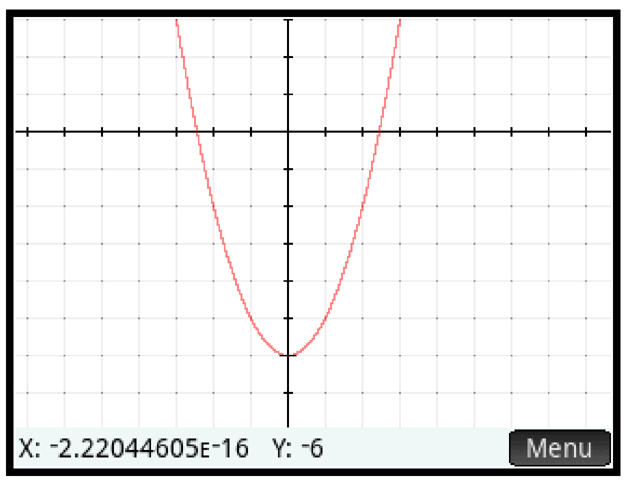
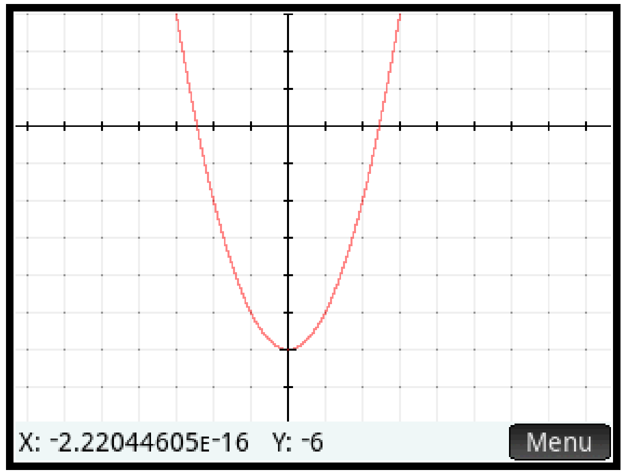


Sketch your horizontal lines here.

16. Let's do the same with a vertical line and a quadratic equation. Keep the quadratic equation from the last question. In V1 enter any vertical line $(x = \_\_)$. Translate your vertical line using different values to determine the types of solutions. Sketch each type of solution on the graph provided.

    What types of solutions did you discover?

    ____



Sketch your vertical lines here.

17. Create your own linear and quadratic system. Enter them below and on the HP Prime. Then sketch your system.

    V1: ______________________________

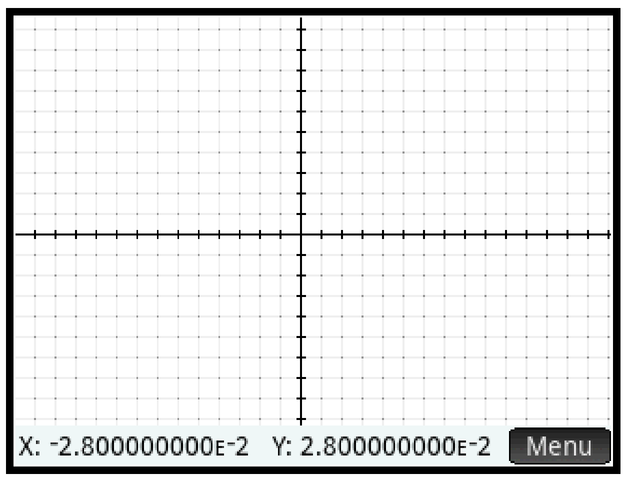
    V2: ______________________________

    How many solutions does your system have?

    ____

    What are the solutions?

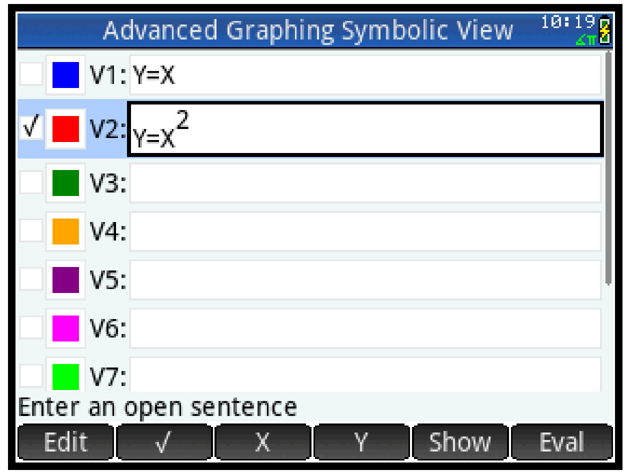
    ____



Sketch your system here.

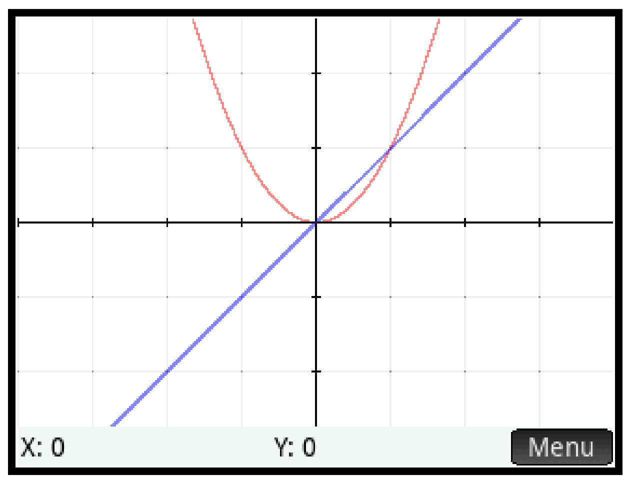
For the Teacher

For this activity, students will use the Advanced Graphing app to investigate a system of linear equations (including horizontal and vertical) and quadratic equations to determine the types of solutions. They will also use the trace feature within the app to find those solutions.



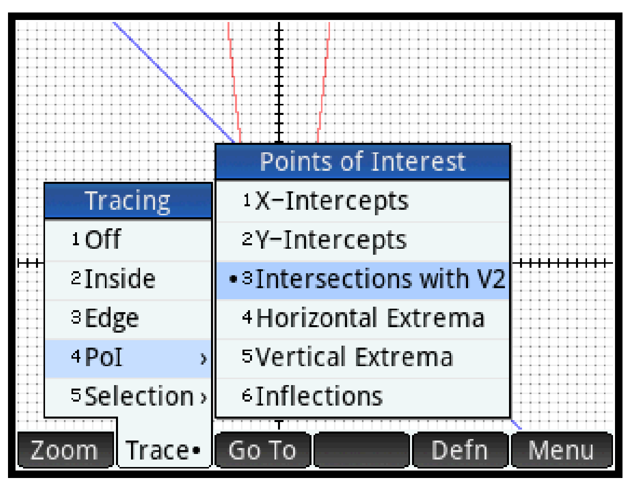
1. HP Prime can help students recognize the number of solutions a system of linear and quadratic equations has by looking at their graphs in plot view.

2. This activity begins by having students find the key features of a linear equation followed by a quadratic equation. This reminds students of the graphical representations of these equations.



3. Students look at the parent functions of a system of linear and quadratic equations and locate their solutions by finding the points of intersection. Students also prove using algebra that these points of intersections are solutions to the system.

4. Students investigate three different systems by graphing the given equations. Students are asked to find the number of solutions. Students will use the trace feature of the app to find the points of intersection.

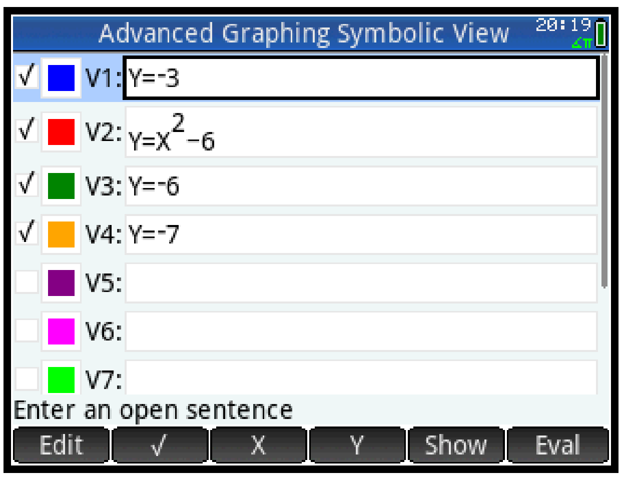


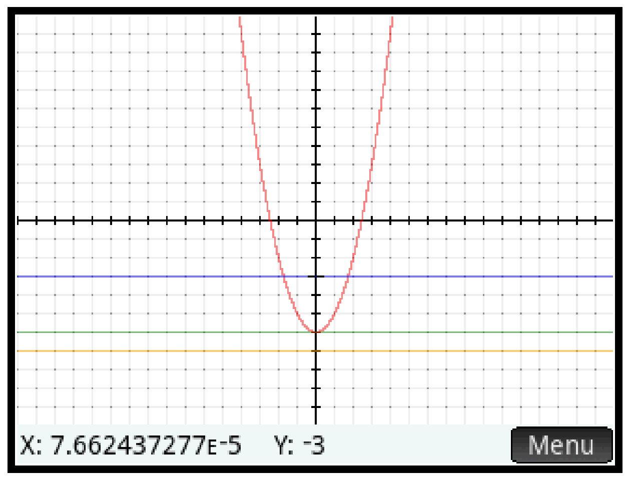
5. Students look at the system of a horizontal/vertical line and a quadratic equation. And repeat the steps above.

6. Finally students are asked to create their own system providing the equations, number of solutions, solution coordinates, and a sketch of their system.

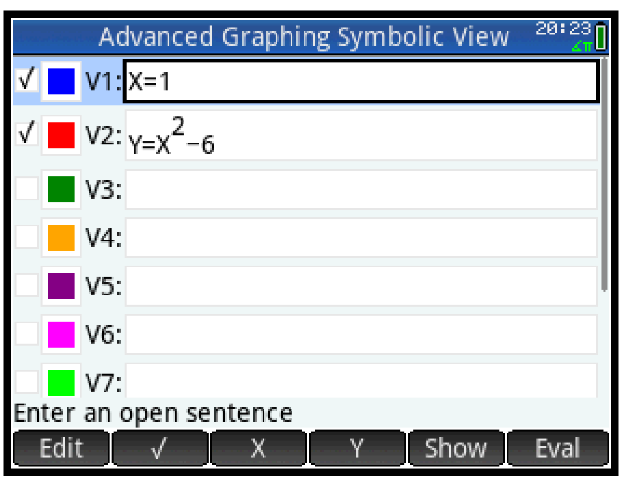
Answer Key (page 1)

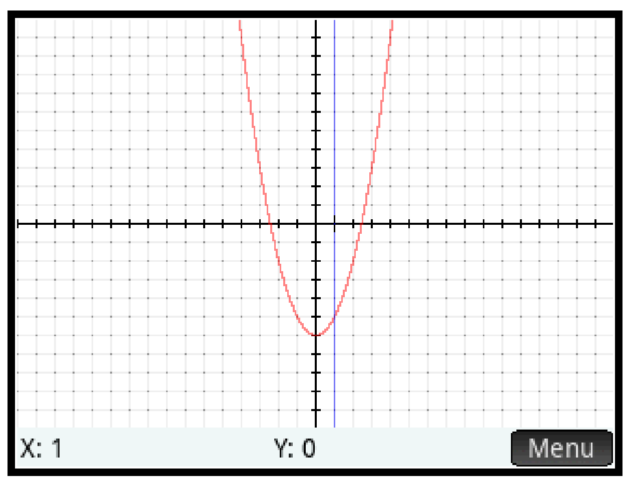
3. Slope: 1
   Y-int: $y = 0$ or $(0,0)$
   X-int: $x = 0$ or $(0,0)$
5. Vertex: $(0,0)$
   Roots: $x = 0$
   Y-int: $y = 0$
6. $(0,0)$ and $(1,1)$
7. $y = x: 1 = 1; 0 = 0$
   $y = x^2: 1 = 1; 0 = 0$
9. 2 solutions
10. $(3,5)$and $(-4,12)$
12. 1 solution at $(1,-3)$
14. 0 solution; none
15. 2 solutions, 1solution, 0 solution





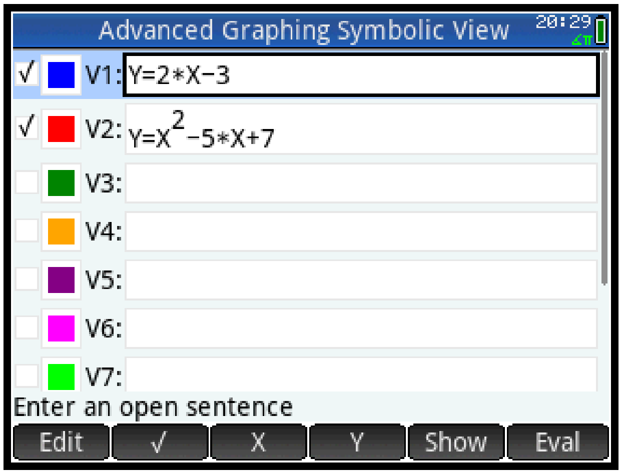
16. 1 solution

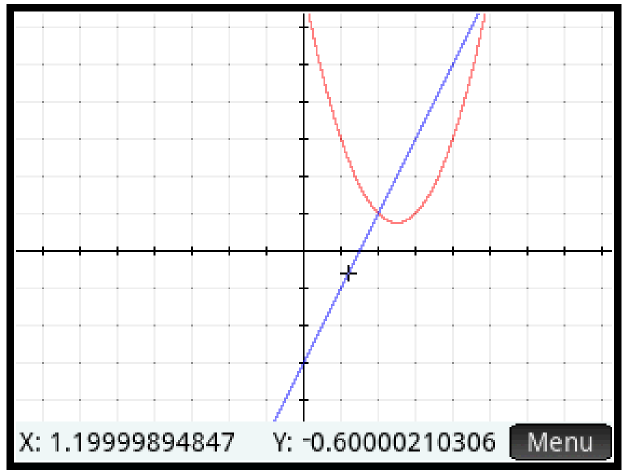




Answer Key (page 2)

17. Student answers will vary. Student should provide a linear equation, quadratic equation, number of solutions and their points of intersection. Students should sketch their system.
    V1: $y = 2x - 3$
    V2: $y = x^2 - 5x + 7$
    Number of solutions: 2 solutions
    Solutions: $(5,7)$ and $(2,1)$





Matching Quadratic Equations and Graphs with HP Prime

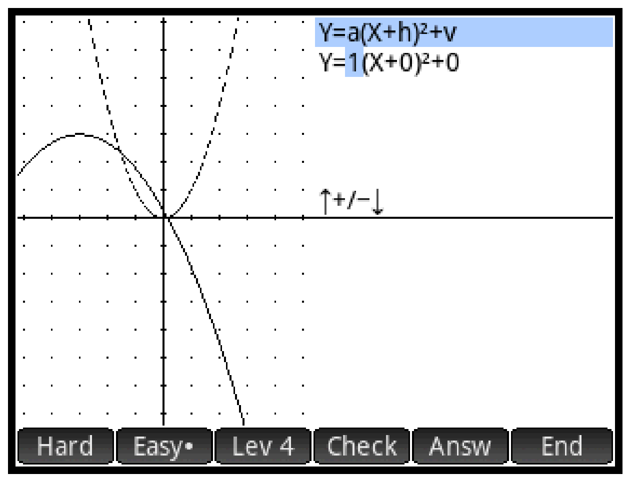
Your goal is to change the values of a, h and v to match the parabola shown. Notice the parent graph will also be shown on the screen with a dotted line. Once you have selected your values press the menu key Check. You will be provided with feedback on if you are correct or incorrect as well as your number of attempts.

This activity will use the following menu keys. To select a menu key tap it on the screen.

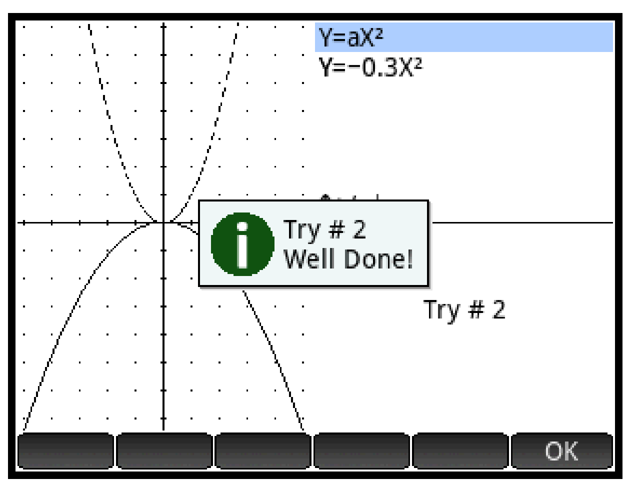
- Test : test feature of app
- Check : check your answer and receive feedback
- Answ: shows the correct answer
- Hard : difficulty level (can also select easy)
- Lev 1 : level selected (1-4)
- OK : Close the window
- End : exit the test feature

For the Student

1. Press Apps Info to open the App Library. Choose the Quadratic Explorer App. The app opens with the graph on the left and the equation on the right. Reset the app by pressing Shift Esc Clear. Tap Test by touching the screen. Graphs are generated by the app and will be different than those represented on this handout.



For questions 2-5.
Match the generated graph by changing the needed variable(s). Once you have made your hypothesis, tap Check. If incorrect try another value until you get it correct. When correct choose OK and then move up in difficulty, tap Hard. Then tap Lev 1 to advance to the next level and tap Easy. If you need help, tap Answ.

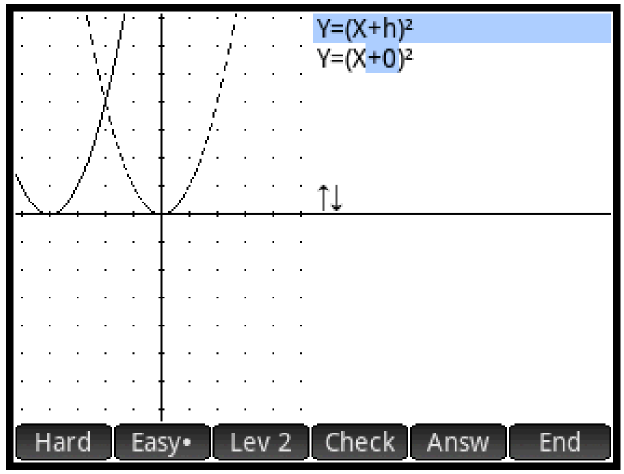


2. Tap Lev 1 until it reads Lev 1. Level 1 focuses only the variable a. You may need to press +/− to change a from positive to negative. Press ▲ ▼ to select your value of a. How does a relate to the graph of a quadratic equation?

____

3. Now select Lev 2. Level 2 focuses only on the variable h. Press ▲ ▼ to select your value for h. How does h relate to the graph of a quadratic equation?

   ____



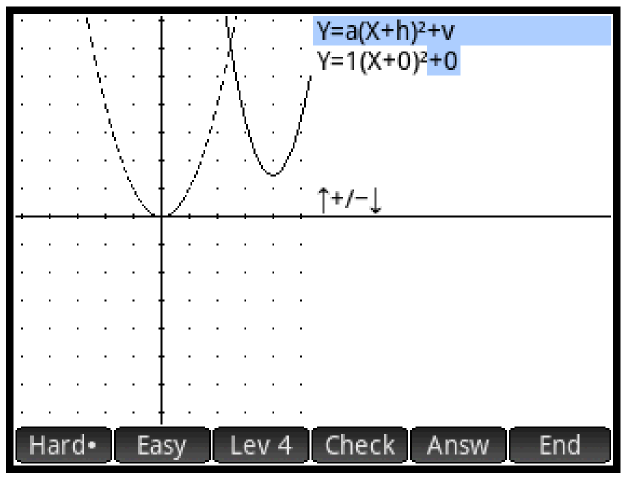
4. Now select Lev 3. Level 3 focuses only on the variable v. Press ▲ ▼ to select your value for v. How does v relate to the graph of a quadratic equation?

   ____

5. Now select Lev 4. Level 4 combines all three variables. Variables may have the value 0. Change each variable to match the given parabola. Record the number of trials needed to correctly match the graph at both difficulty levels.

   Easy: ______________________________

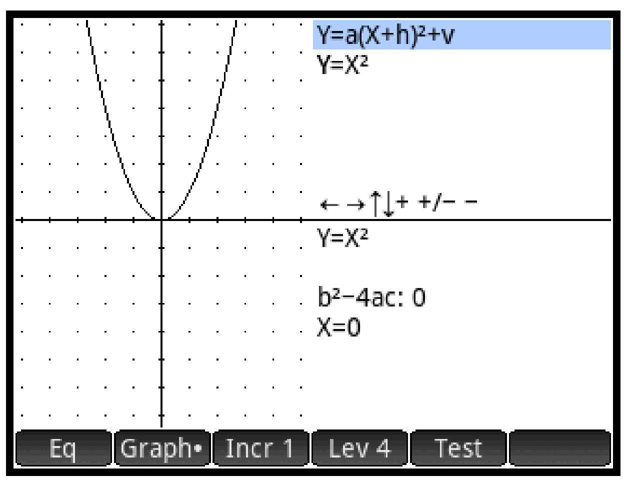
   Hard: ______________________________



6. Write an equation that has a different value for a, h and v. Sketch its graph on the screen shown on the right.

Equation:______________________________

   What steps did you take to graph your equation?



Sketch your graph.

7. For more practice, keep playing.

For the Teacher

For this activity, students will use the Test feature of the Quadratic Explorer App to apply their understanding of how variables $a, h$and v relate to the graph. HP Prime app will generate a variety of parabolas using 4 levels and two difficulties.

1. HP Prime can help students understand the properties of a quadratic equation and its graph through the Quadratic Explorer app. Students explore the variables a, h and v in the equation $Y = a(x - h)^2 + v$.

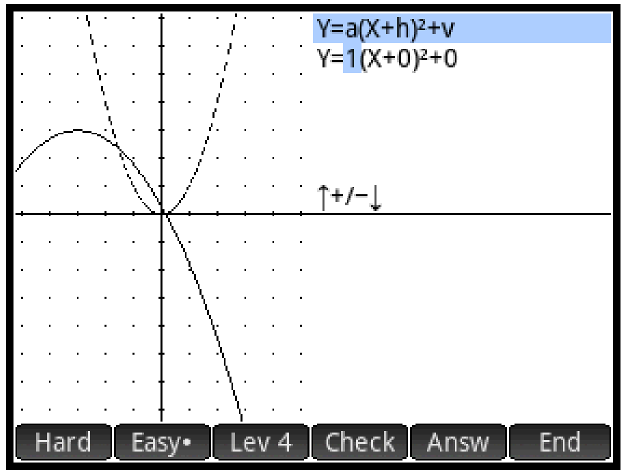
2. This activity begins by having students play the Test game starting with Level 1 Easy. Once the student has found the correct answer he/she moves to Hard. Once both difficulties have been solved he/she moves to the next level.

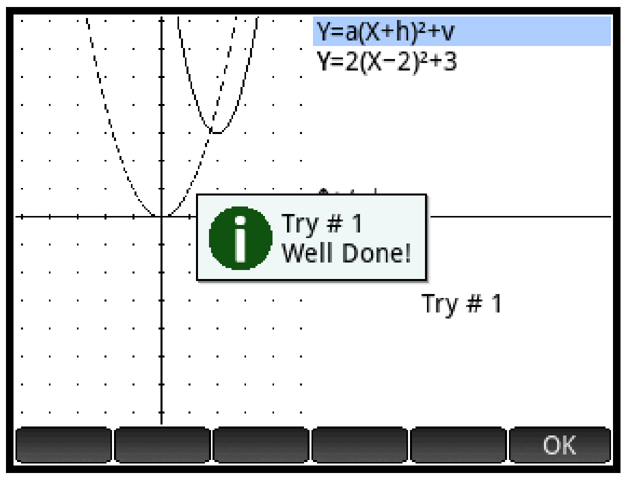
Each level has a different focus.
Lev 1: Quadratic Coefficient (a)
Lev 2: Horizontal Shift (h)
Lev 3: Vertical Shift (k)
Lev 4: Combination (a, h, k)

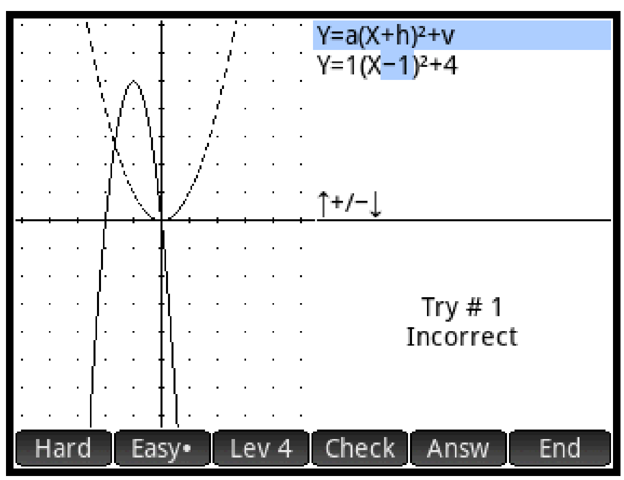
3. Once students have selected their values they can tap Check to receive immediate feedback from HP Prime. This feedback tells the number of attempts and whether the student is correct (Well done!) or incorrect.

4. Finally students will be asked to create their own quadratic equation, sketch its graph, and provide their steps.

Note: The answer can also be given for students as needed by selecting Answ from the menu bar.

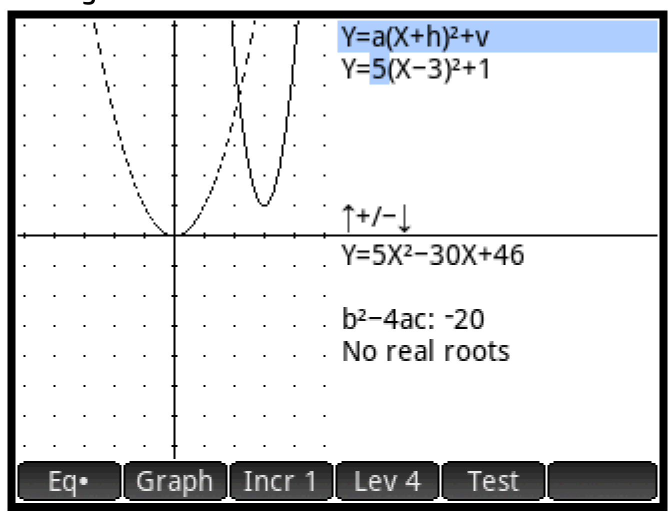






Answer Key

2. a gives the direction. Up: $a > 0$ Down: $a < 0$
   a gives the dilation Stretch: $|a| > 1$ Compress: $0 < |a| < 1$
3. h gives the horizontal shift or x-value of the vertex
4. v gives the vertical shift or y-value of the vertex
5. Answers Vary: Students should give the amount of trials needed at each difficulty level.
   Easy: 2
   Hard: 3
6. Answers Vary: Students should write an equation, sketch its graph, and explain their steps.
   Equation: $y = 5(x-3)^2 + 1$
   I found the vertex $(3, 1)$ and plotted a point. Next I looked at the a value it is positive so my parabola opens up. It is larger than 1 so it is being stretched.



HP PRIME GRAPHING CALCULATOR
CONTENT

R

REVIEWS & ADVERTS

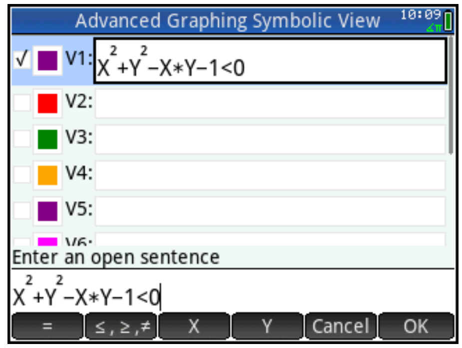
A Prime Graphed Rose For You

G.T. Springer

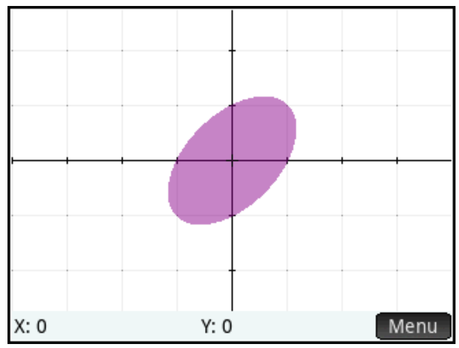
In this short article, we discuss how to use the HP Prime Advanced Graphing app to teach students about transformations in the plane, including translations and reflections. But instead of a geometric approach, the app allows us to take an algebraic approach. We start with a generic shape, which we then transform in various ways to build a picture of a rose.

We start by defining our generic shape, a filled ellipse, given by $x^2 + y^2 - x \cdot y - 1 < 0$.

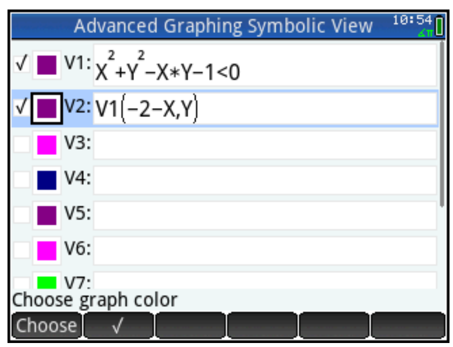
- Press  and tap the *Advanced Graphing* icon
- Enter the expression above into V1, using  and  to help you
- Tap on the color picker for V1 and choose purple



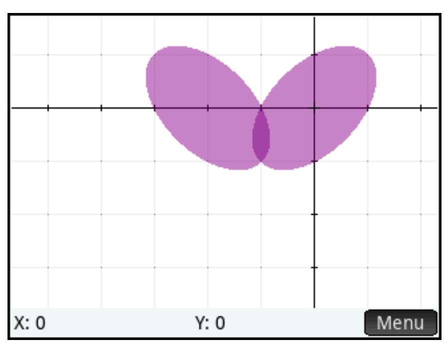
Press  to see the graph of our ellipse. Press + to zoom in. This is the first "petal" of our rose.



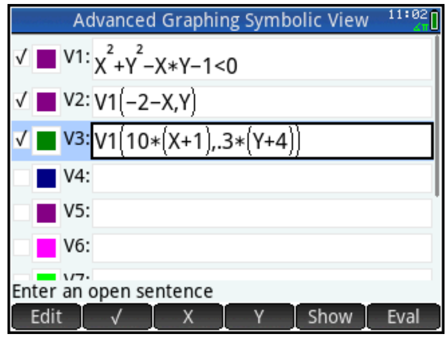
A reflection over the vertical line x=-1 is a mapping such that *x*→-2-*x* and *y*→*y*. So we define V2 to be V1(-2-x, y), as shown in the figure to the right. We use the same color (purple) for our reflection.



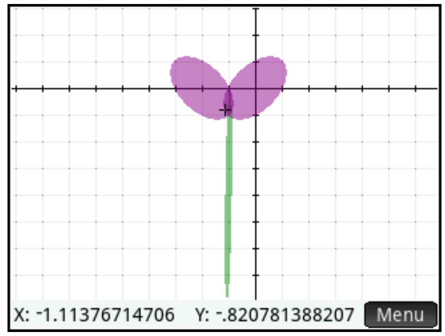
Press  to see the second "petal"; tap and drag to scroll the view.



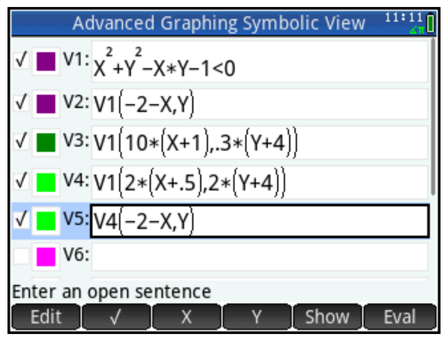
We need a stem for our rose. Creating a stem takes some experimentation. Students will learn to stretch the base shape first, then translate it into its final position. We defined V3 to be V1(10*(X+1),0.3*(Y+4)), as shown to the right. Do not forget to change the color to dark green!



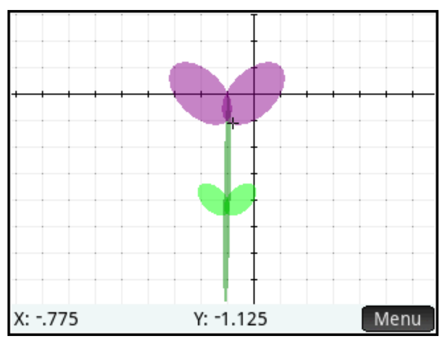
Again, press  to see the rose starting to take shape. Press w to zoom out and drag to scroll until you get the view you want.



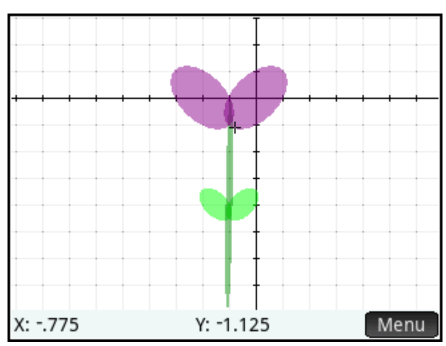
Next, we add some leaves. For the first leaf (in V4), we experiment as we did for the stem, first dilating and then translating to get what we want. Then the second leaf is the reflection of the first leaf, using the same method we used for the petals. We chose light green as the color for both leaves.



And now you have a simple rose!



For extra credit, we added two fancy petals and some thorns. The expressions are shown on the next page in case you want to work them out for yourself first!



Be sure to save your new app. Press Apps Info to go to the App Library; the Advanced Graphing app should already be highlighted. Tap Save, enter a name for your new app (we used ARose4You), and tap OK twice. The new app now appears in your App Library.

Getting the most out of your HP Prime Virtual Calculator for the PC.

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If you are using the HP Virtual Prime for the PC (it is on the product CD that came with your HP Prime), you can just copy and paste the expressions below into the virtual calculator. Then you can send the app from the virtual calculator to your physical HP Prime. The steps are listed below.

Advanced Graphing app expressions for the rose

V1: X^2+Y^2-X*Y-1<0
V2: V1(-2-X,Y)
V3: V1(10*(X+1),.3*(Y+4))
V4: V1(2*(X+.5),2*(Y+4))
V5: V4(-2-X,Y)
V6: Y+1>(X+1)^2 AND Y<1+COS(2*X+3)
V7: V6(-2-X,Y)
V8: V1(20*(X+.8),10*(Y+3))
V9: V8(-2-X,Y)

To copy the rose expressions to your HP Prime Virtual Calculator

1. Select and copy the expression named V1. Note: select and copy everything <u>after</u> “V1:”
2. On the HP Prime Virtual Calculator, go to the Rose app Symbolic view and select the field **V1**
3. In the menu bar, click on *Edit* and select *Paste*. The expression will be pasted into the V1 field.
4. Tap OK to accept the expression. It will now graph properly.
5. Repeat Steps 2-4 for the other expressions (V2-V9)

To send the Advanced Graphing app from your virtual calculator to a physical HP Prime

1. Connect your physical HP Prime to your PC using the USB cable
2. Turn on your HP Prime and launch the HP Prime Virtual Calculator for the PC
3. On the virtual Prime menu bar, click *Calculator*, then click *Connect To* and select your physical Prime
4. Go to the App Library, select the Rose app, and tap Send
5. The Rose app is now on your physical Prime and ready for viewing!

HP PRIME GRAPHING CALCULATOR

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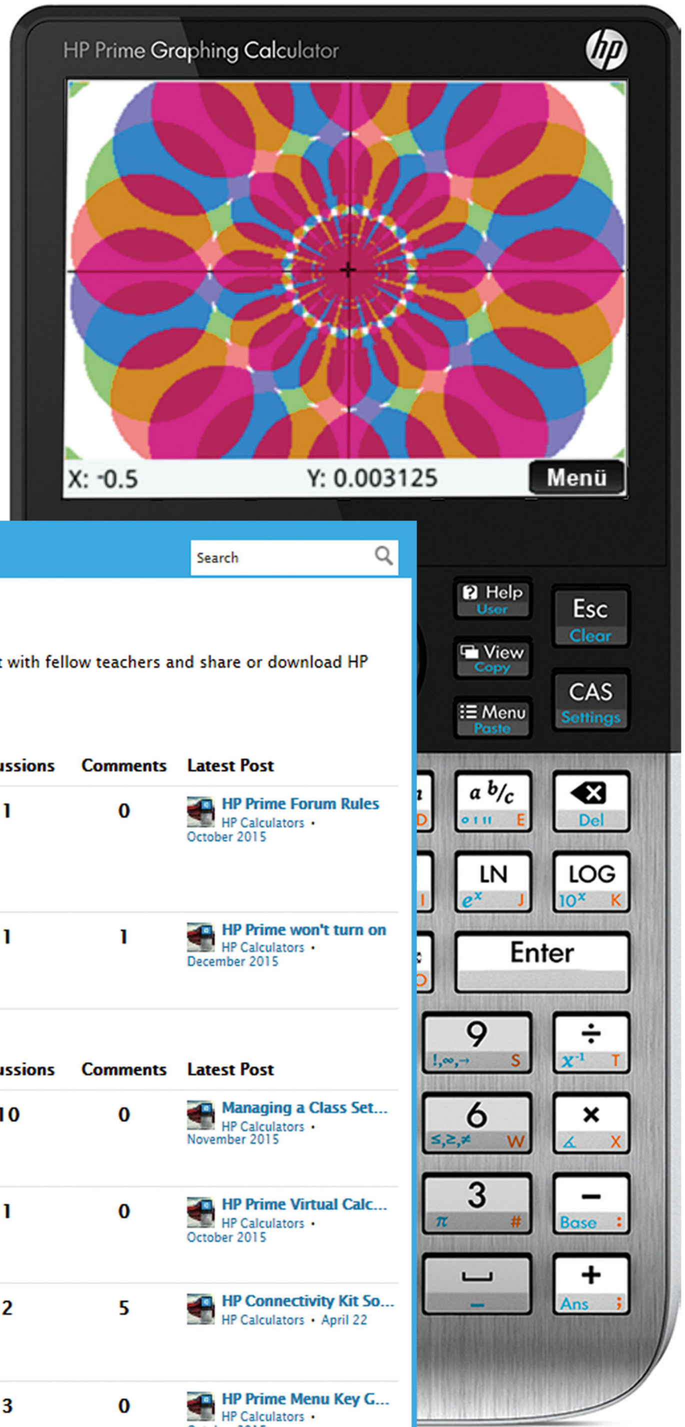

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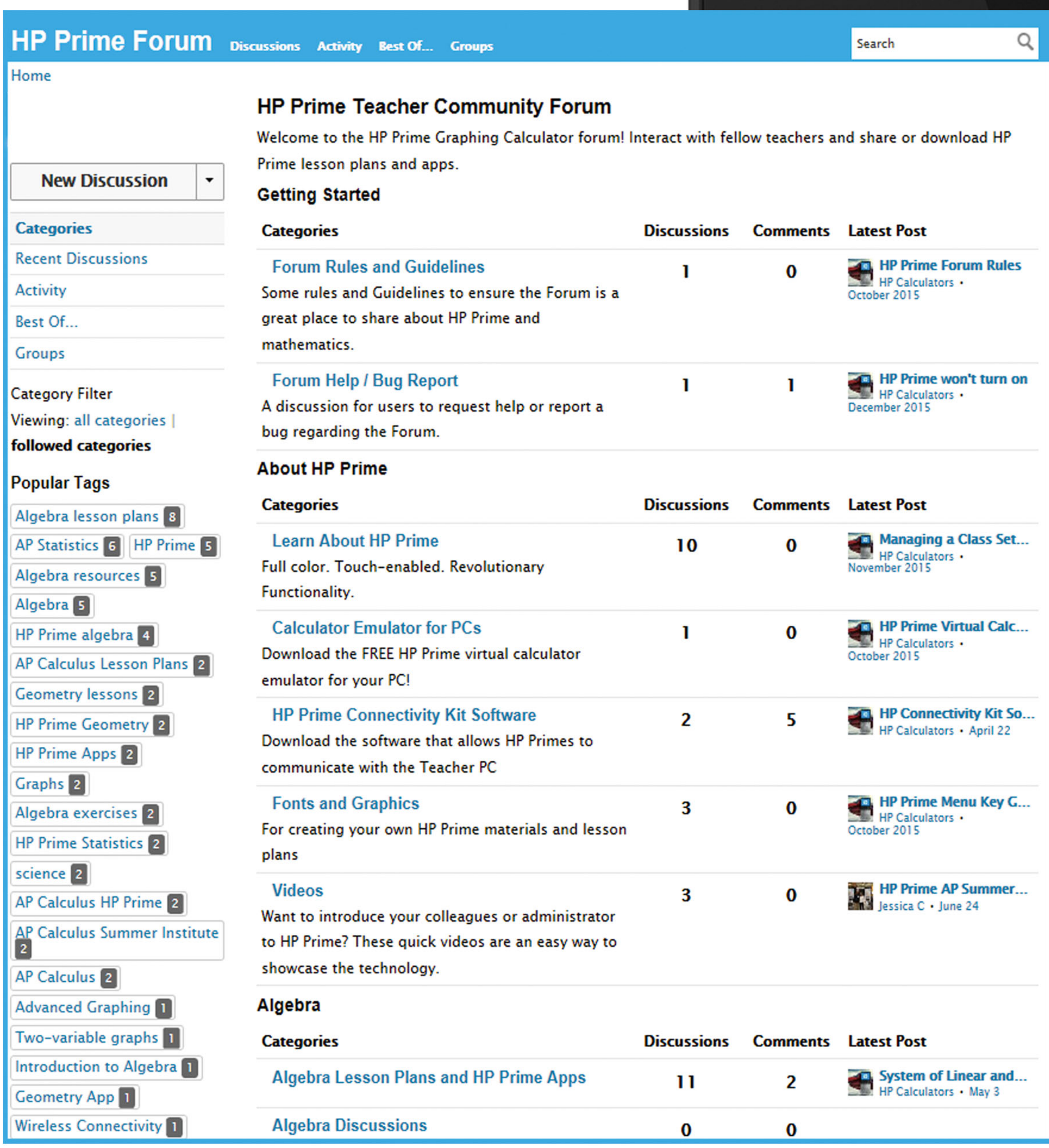
- Laden Sie Apps und Programme herunter
- Tauschen Sie Erfahrungen aus
- Stellen Sie die eigene Programme zur Verfügung

Diskutieren Sie mit !

www.hp4calc.ch/prime



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HP Prime Teacher Community Forum

Welcome to the HP Prime Graphing Calculator forum! Interact with fellow teachers and share or download HP Prime lesson plans and apps.

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